Abstract

Labour costs in various European countries have reached a record high in recent years. The topic of non-wage labour costs is therefore increasingly being discussed among and between the political parties because non-wage labour costs are likely to have major negative effects on employment. We follow the real options approach, which allows us to investigate the value to a firm of waiting to adjust labour when the firm’s wage and non-wage costs are stochastic and adjustment costs are sunk. Simulation exercises show that the interaction between hiring and firing costs, non-wage labour costs and uncertainty can have important ramifications for employment dynamics.

I Introduction

Non-wage labour costs are the subject of intensive political debate. Payroll taxes drive a wedge between the cost of a worker to an employer and the wage received. If wages and prices are relatively flexible, high non-wage labour costs are unlikely to have major negative effects on employment in the long-run. However, in countries where wages and prices are inflexible, employment will suffer if non-wage labour costs increase. Many of the job losses will fall on low-paid workers, due among other things to the existence of binding wage floors such as legal or collectively-bargained minimum wages. Increasing non-wage labour costs also tend to encourage substitution away from labour to more capital-intensive methods of production. Therefore, reducing social insurance contributions ranks high on the European political agenda.

Non-wage labour costs are those categories of the enterprise’s total labour costs comprising other than direct compensation. Non-wage labour costs account for a very substantial and rising proportion of total labour costs. There are several ways of defining non-wage labour costs. The annual analysis of non-wage labour costs in Germany by the Institut der deutschen Wirtschaft,
e.g. distinguishes between compensation for hours actually worked and non-wage labour costs. Non-wage labour costs are differentiated into pay for days not worked, special payments, statutory social welfare costs and other non-wage labour costs. Table 1 below sets out the latest aggregate data on the development of wage and non-wage labour costs in West German industry [see Schröder (2004)].

In West German industry, non-wage labour costs reached an all-time high of €22,350 in 2003. From 1972 to 2003, the ratio of non-wage labour costs to direct compensation grew by 22.6 percentage points to 78.2%.

Table 2 indicates that there is significant variation in wage and non-wage labour costs across countries. Hourly non-wage labour costs in West German industry amounted to €11.62 in 2002. This was above of all countries compared. There is widespread agreement in high non-wage cost countries that non-wage labour costs are far too high and have to be reduced because they drive up labour costs and thus reduce the demand for labour, particularly for hard-to-place workers. Firms also claim that uncertainty about the future level of non-wage labour costs is an impediment to job creation. Therefore, they form expectations and beliefs on the future behaviour of the driving economic variables, which cannot be predicted with certainty. The modelling framework has to account for this distinct challenge and has to formalise this issue in a coherent economic model.

Orthodox theory suggests calculating the net present value (NPV) of a mooted employment decision. When the present value of future profits is bigger than the present value of the costs of hiring a worker – that is, the NPV is positive – then go ahead. All employment calculations therefore rely on predicting uncertain future profits. But the traditional theory also assumes, implicitly, that employment decisions are a now-or-never choice. In many circumstances this is unrealistic and waiting offers a valuable chance to learn more about the likely fate of the decision. The ability to delay a partially irreversible employment decision is like a financial ‘call option’. The firm has the

1 After reunification, the entire German welfare system was transferred to the East, including the idea of wage equalisation. Naturally, a huge part of the East German economy could not create the necessary productivity, soon resulting in high unemployment in eastern Germany. As, basically, the huge amounts of social transfers had to be borne by western workers and employees, the government has been forced to increase payroll and non-wage labour costs, which further spurred the wage-unemployment circle.

2 In East German manufacturing industry, annual direct compensation (non-wage labour costs) reached €20,160 (€13,480) in 2003 and therefore the share of non-wage labour costs reached 67% of total compensation. Though rising, non-wage labour costs in eastern Germany were still lower than in western Germany. The differences result from less generous fringe benefits such as vacation and supplementary pension schemes.

3 An interesting feature is that some European countries succeeded to restore lower rates of unemployment in the 1990s despite a high share of non-wage labour costs. A remarkable example for the way out of Europe’s labour market misery is the Netherlands. The Dutch employer associations and unions reached a historic agreement on wage moderation (the so-called Wassenaar Agreement) in 1982. It turned out that wage moderation was an essential ingredient of the Dutch success story. The argument is typically that central bargaining, which incorporates the leapfrogging externality incorporated in industry-level bargaining will yield lower rates of unemployment for a given rate of economic growth.

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right, but not the obligation, to buy (hire) a security (new employees) at a specified price (the hiring cost) at a future time of its choosing. This option has a value. When the firm makes the investment, it exercises (or, in financial jargon, ‘kills’) its option. It follows then, that the cost of that ‘killed’ option (the value of

Table 1
Annual labour costs in West German industry per employee (€)

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</tr>
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<tbody>
<tr>
<td>Direct</td>
<td>7535</td>
<td>9600</td>
<td>11,557</td>
<td>13,616</td>
<td>15,406</td>
<td>17,580</td>
<td>21,314</td>
<td>24,218</td>
<td>25,000</td>
<td>26,455</td>
<td>28,580</td>
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<tr>
<td>compensation</td>
<td>4188</td>
<td>6304</td>
<td>8099</td>
<td>10,276</td>
<td>12,198</td>
<td>14,149</td>
<td>17,139</td>
<td>19,852</td>
<td>20,450</td>
<td>21,505</td>
<td>22,350</td>
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<tr>
<td>Non-wage</td>
<td>55.6</td>
<td>65.7</td>
<td>70.1</td>
<td>75.5</td>
<td>79.2</td>
<td>80.5</td>
<td>80.4</td>
<td>82.0</td>
<td>81.8</td>
<td>81.3</td>
<td>78.2</td>
</tr>
<tr>
<td>labour costs</td>
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<td>Share of</td>
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<td>non-wage</td>
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<tr>
<td>labour costs (%)</td>
<td>55.6</td>
<td>65.7</td>
<td>70.1</td>
<td>75.5</td>
<td>79.2</td>
<td>80.5</td>
<td>80.4</td>
<td>82.0</td>
<td>81.8</td>
<td>81.3</td>
<td>78.2</td>
</tr>
</tbody>
</table>

Notes:
The calculations are based on official statistics from the Federal Statistical Office in Wiesbaden, which conducts surveys on labour costs every 4 years. The Institut der Deutschen Wirtschaft has extrapolated these official statistics to 2003, based on several additional statistics.

Table 2
International comparison of hourly industrial wages, 2002 (€)

<table>
<thead>
<tr>
<th></th>
<th>Total hourly wage costs</th>
<th>Direct hourly wages</th>
<th>Non-wage labour costs</th>
<th>Share of non-wage labour costs (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>21.64</td>
<td>11.19</td>
<td>10.45</td>
<td>93</td>
</tr>
<tr>
<td>Belgium</td>
<td>23.35</td>
<td>12.22</td>
<td>11.12</td>
<td>91</td>
</tr>
<tr>
<td>Canada</td>
<td>17.44</td>
<td>12.58</td>
<td>4.86</td>
<td>39</td>
</tr>
<tr>
<td>Denmark</td>
<td>25.73</td>
<td>19.64</td>
<td>6.09</td>
<td>31</td>
</tr>
<tr>
<td>Finland</td>
<td>23.20</td>
<td>13.05</td>
<td>10.15</td>
<td>78</td>
</tr>
<tr>
<td>France</td>
<td>19.50</td>
<td>10.20</td>
<td>9.30</td>
<td>91</td>
</tr>
<tr>
<td>East Germany</td>
<td>16.43</td>
<td>9.96</td>
<td>6.47</td>
<td>65</td>
</tr>
<tr>
<td>West Germany</td>
<td>26.36</td>
<td>14.74</td>
<td>11.62</td>
<td>79</td>
</tr>
<tr>
<td>Greece</td>
<td>9.47</td>
<td>5.64</td>
<td>3.82</td>
<td>68</td>
</tr>
<tr>
<td>Ireland</td>
<td>17.17</td>
<td>12.29</td>
<td>4.88</td>
<td>40</td>
</tr>
<tr>
<td>Italy</td>
<td>16.60</td>
<td>8.53</td>
<td>8.08</td>
<td>95</td>
</tr>
<tr>
<td>Japan</td>
<td>20.18</td>
<td>12.06</td>
<td>8.12</td>
<td>67</td>
</tr>
<tr>
<td>Netherlands</td>
<td>22.64</td>
<td>12.63</td>
<td>10.01</td>
<td>79</td>
</tr>
<tr>
<td>Norway</td>
<td>28.52</td>
<td>19.20</td>
<td>9.31</td>
<td>49</td>
</tr>
<tr>
<td>Portugal</td>
<td>6.59</td>
<td>3.74</td>
<td>2.84</td>
<td>76</td>
</tr>
<tr>
<td>Spain</td>
<td>15.37</td>
<td>8.42</td>
<td>6.96</td>
<td>83</td>
</tr>
<tr>
<td>Sweden</td>
<td>21.86</td>
<td>12.90</td>
<td>8.97</td>
<td>70</td>
</tr>
<tr>
<td>Switzerland</td>
<td>26.24</td>
<td>17.20</td>
<td>9.03</td>
<td>53</td>
</tr>
<tr>
<td>UK</td>
<td>19.89</td>
<td>13.76</td>
<td>6.14</td>
<td>45</td>
</tr>
<tr>
<td>US</td>
<td>22.44</td>
<td>16.18</td>
<td>6.26</td>
<td>39</td>
</tr>
</tbody>
</table>

Source:
waiting for better information) ought to be included when calculating the NPV. Before a hiring decision goes ahead, the present value of future profits should exceed the hiring costs by at least the value of keeping the real option alive. In other words, real options are directly analogous to a traditional American call option. While real options are similar, the primary distinction is the non-financial nature of the underlying asset being acquired.4

Against this background, the paper proceeds as follows. The application of the real options approach to employment determination is sketched in Section II. Section III discusses the simulation results. Finally, Section IV concludes with a few remarks. Two appendices provide technical results used in the body of the paper.

II LABOUR DEMAND AND NON-WAGE LABOUR COSTS IN A REAL OPTIONS FRAMEWORK

We consider a representative firm facing the constant returns to scale Cobb–Douglas production function

\[ Y = K^a L^{1-a}, \] (1)

where \( Y \) denotes output, \( 0 < a < 1 \) is the distribution parameters, \( L \) is the number of employees, and \( K \) is the capital stock. We allow for imperfect competition, i.e. we assume that the firm faces an isoelastic demand function

\[ p = Y^{(1-\psi)/\psi} Z, \quad \psi \geq 1, \] (2)

where \( p \) represents the price, \( Z \) is a parameter, and \( \psi \) is an elasticity parameter that takes its minimum value of 1 under perfect competition. Therefore, current profits, measured in units of output, are defined as

\[ \Pi = ZK^\tau L^{1-a} - w(1+\tau)L, \] (3)

where \( \tau \) denotes the ratio of non-wage labour costs to the stochastic wage-related costs, \( w \). To keep the model simple we abstract from taxes other than those included in the non-wage labour costs. The firm faces input cost uncertainty. Following Pindyck (1993), this notion is formalised by assuming that direct wage costs follow a geometric Brownian motion without a drift:

\[ dw = \sigma w dW - \eta w \frac{W}{\tau} d\tau, \] (4)

where \( W \) is a Wiener process, \( dW_t = \varepsilon_t \sqrt{dt} \) (\( \varepsilon_t \) is a random variable drawn from a standardised normal distribution with zero mean and unit variance and \( \varepsilon_t \) is serially uncorrelated due to the assumption of independent increments of the

4 Bowman and Maskowitz (2001, p. 777) have recently concluded that the real options approach 'encourages experimentation and the proactive exploration of uncertainty' and thus a 'revolution in thinking'. Surveys of the real options literature are provided by Amran and Kulatilaka (1999), Copeland and Antikarov (2001), Coy (1999), Dixit and Pindyck (1994) and Lander and Pinches (1998). Recent applications of the real options approach in the labour demand literature include Booth et al. (2002), Chen and Funke (2004) and Chen and Cheng (2005).
Wiener process), and $\sigma$ is the variance parameter. Equation (4) allows for partial shifting of the burden of payroll taxes from employers to workers. We assume that an increase (decrease) in non-wage labour costs, $dt$, leads to a decrease (increase) of wages by a certain percentage ($\eta$) of the tax change. Although the aim of this paper is not to provide an in-depth presentation of existing and sometime conflicting bargaining models, equation (4) incorporates non-wage labour costs into the wage setting. A cursory look at the main features of different models may help understanding the interaction between the tax burden on labour and wages. While the statutory incidence of a tax may be relevant for political reasons it is well known from tax theory that the statutory incidence may give a rather inaccurate picture of the economic incidence of payroll taxes. To the extent that the price of the item taxed changes when a tax is levied, the tax is shifted and the final incidence can be different from that implied by the statutory nominal incidence. The actual burden of a tax depends on a set of complicated behavioural responses and generally falls on the side of the market that is most inelastic. Moreover, the impact of labour taxes upon wages also depends on the interactions with other institutions (e.g., the nature of wage negotiations, the degree of real wage rigidity, and the availability of unemployment benefits). On the other hand, hiring and firing costs give incumbent workers some market power. The potential for incumbent workers to extract rents from hiring and firing costs was therefore used by Lindbeck and Snower (1988) to explain hysteresis in unemployment. The ideal would therefore be a fully dynamic model, which allows understanding the complex and multifaceted issues surrounding endogenous wage determination. Unfortunately, such models have thus far proved intractable and remain a difficult task.\(^5\)

Given this situation, equation (4) provides a flexible but nevertheless thorough reduced-form specification. Although it is stylised and does not capture all of the details it nevertheless allows for a partial shifting of the burden of payroll taxes from employers to workers.

The representative risk-neutral firm maximises its discounted flow of profits

$$V = \max_{L} E \left[ \int_{0}^{\infty} \left[ ZK^{s}L_{s}^{1+\delta} - w_{s}(1+\tau_{s})L_{s} \right] e^{-rs} ds \right|_{w_{0} = w, \ \tau_{0} = \tau, \ \L_{0} = L},$$

(5)

where $E[\cdot]$ is the expectation operator, $V$ denotes the intertemporal profit function, and $r$ is the real interest rate. We assume that employees quit at an exogenous rate $\delta$. It is assumed that the payroll tax $\tau$ follows the following two jump processes

$$d\tau = dJ_{1} + dJ_{2},$$

(6)

where $dJ_{1}$ and $dJ_{2}$ are the increments of Poisson processes (with mean arrival rates $\lambda_{1}$ and $\lambda_{2}$). It is assumed that if an `event 1’ (`event 2’) occurs, $\tau$ increases

\(^5\)Hamermesh and Pfann (1996, p. 1270), e.g. comment that a departure from the assumption of a fixed wage leads to substantial complications.

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(falls) by $\phi_1$ ($\phi_2$) percent with probability 1.\textsuperscript{6} Over each time interval $dt$ there is a probability $\lambda_1 dt$ (or $\lambda_2 dt$) that it will rise (drop) by $\phi_1 \tau$ ($\phi_2 \tau$). Additionally, we assume that $(dJ_1 dJ_2)$ and $dW$ are orthogonal, i.e. $E(dW dJ_1) = 0$, $E(dW dJ_2) = 0$, and $E(dJ_1 dJ_2) = 0$.

Equations (4) and (6) indicate that there are two sources of uncertainty. Following Pindyck (1993), type I uncertainty represented by the geometric Brownian motion captures direct wage cost uncertainty. Instability of this type may be helpful in predicting the variability in profits. To understand the policy impact upon labour demand, we have additionally assumed type II uncertainty (represented by the two jump processes). This newly added uncertainty represents political uncertainty about future changes in non-wage labour costs and allows investigating how uncertainty about future non-wage labour costs alters incentives for employment.\textsuperscript{7} In our work, the timing of the potential policy shifts is exogenous.\textsuperscript{8} In other words, our model contains two uncorrelated jumps and the behaviour between the jumps is that of a ‘Gaussian’ diffusion (‘Poisson–Gaussian model’). The critical question for the firm is how best to respond in such an uncertain environment.\textsuperscript{9} Using Itô’s Lemma, the Bellman equation for the value $V$ at time zero is

$$
rV = \max_L \left\{ ZK^2L^\frac{1+\mu}{\gamma} - w(1 + \tau)L - \delta LV_L + \frac{1}{2}\sigma^2w^2V_{ww} + \lambda_1 [V(w(1 - \eta \phi_1), (1 + \tau(1 + \phi_1))) - V(w, 1 + \tau)] + \lambda_2 [V(w(1 + \eta \phi_2), (1 + \tau(1 - \phi_2))) - V(w, 1 + \tau)] \right\}.
$$

To find the optimal condition for employees with the existence of firing costs and hiring costs, we need to obtain the value of the marginal employed worker first ($v = VL$) and then compare the marginal value of employees with the marginal hiring and firing costs. We take the derivative of (7) with respect to $L$.

\textsuperscript{6}Given the high level of unemployment, the German government has implemented an environmental tax reform in several stages since 1999 and has used the proceeds to cut employer and employees contributions to the pension fund. This policy-induced fall of non-wage labour costs may be represented by ‘event 2’ in equation (6). A reduction of payroll taxes paid by employers can enhance the international competitiveness of a country, thereby acting like a real exchange rate depreciation. This effect has been called the ‘internal exchange rate depreciation’ in the Scandinavian policy debate [Alesina and Perotti (1997) and Calmfors (1998)].

\textsuperscript{7}Like Merton (1976), the Brownian motion part describes the instantaneous part of the unanticipated normal input price change, while the Poisson processes describe the part due to abnormal input cost changes.

\textsuperscript{8}One potential criticism concerns the manner in which policy is treated in the model. A higher level of factor demand may encourage the government to change policies in the future, so that a firm anticipating this kind of time inconsistency will tend to be self-protective, assuming that the jump parameter is positively correlated with the stock of labour and therefore an endogenous variable. See Cherian and Perotti (2001) for a discussion of the effects of strategic interactions drawing inspirations from the time-inconsistency literature.

\textsuperscript{9}Following the standard modelling approach, we study investment in a single factor and therefore implicitly assume that no other factors are present or that they can be instantaneously and costlessly adjusted. Optimal stopping models with interrelated factor demand decisions are analysed in Eberly and van Mieghem (1997) and Dixit (1997).
and let $v_L = V_{LL}$ and $v_{ww} = V_{Lww}$,

$$(r + \delta)v = L^{\frac{1}{\delta^2} - 1}F(K) - w(1 + \delta) - \delta L v_L + \frac{1}{2} \sigma^2 w^2 v_{ww} + \lambda_1 [v(w(1 - \eta \phi_1), (1 + \tau(1 + \phi_1))) - v(w, 1 + \tau)] + \dot{\lambda}_2 [v(w(1 + \eta \phi_2), (1 + \tau(1 - \phi_2))) - v(w, 1 + \tau)].$$

(8)

and

$$F(K) = \frac{1 - a}{\psi} ZK^2.$$

(9)

The solution for $v(w, \tau)$ consists of the particular integral and the complementary function. We first deal with the identification of uncertainty effects in the very special case where the firm never hires or fires employees. This special case turns out to be useful as a starting point and for comparisons. Then we turn to the general case with positive hiring and firing costs. In the absence of hiring and firing, the particular integral may be expressed as

$$v^P(w) = E \left[ \int_0^\infty \left[ F(K) L^{\frac{1}{\delta^2} - 1} - w(1 + \tau_1) \right] e^{-(r + \delta) s} ds \bigg| w_0 = w, \tau_0 = \tau, L_0 = L \right],$$

(10)

which is the expected present value of the marginal employed worker. This integral can be rewritten as (a proof is given in Appendix A)

$$v^P(Z) = \frac{F(K) L^{\frac{1}{\delta^2} - 1}}{r + (1 - \alpha) \delta / \psi - \frac{w}{r + \delta + \eta(\dot{\lambda}_1 \phi_1 - \dot{\lambda}_2 \phi_2)}} - \frac{w}{r + \delta - \dot{\lambda}_1 \phi_1 (1 - \eta - \eta \phi_1) + \dot{\lambda}_2 \phi_2 (1 - \eta + \eta \phi_2)}.$$  

(11)

The firm’s option value of hiring in the future and its option value of firing once the worker is employed are measured by the complementary function:

$$(r + \delta)v = - \dot{\lambda} L v_L + \frac{1}{2} \sigma^2 w^2 v_{ww} + \dot{\lambda}_1 [v(w(1 - \eta \phi_1), (1 + \tau(1 + \phi_1))) - v(w, 1 + \tau)] + \dot{\lambda}_2 [v(w(1 + \eta \phi_2), (1 + \tau(1 - \phi_2))) - v(w, 1 + \tau)].$$

(12)

Letting $v^G$ be the value of the option, the general solutions have the following forms (see Appendix B for details):

$$v^G = A_1[w(1 + \tau)]^{\beta_1} + A_2[w(1 + \tau)]^{\beta_2},$$

(13)

where $\beta_1$ and $\beta_2$ are the positive and negative roots of the following characteristic equation

$$\frac{1}{2} \sigma^2 \beta(\beta - 1) + \dot{\lambda}_1 \left[ \frac{1 + \tau(1 + \phi_1)}{1 + \tau} \right]^{\beta} (1 - \eta \phi_1)^{\beta} + \dot{\lambda}_2 \left[ \frac{1 + \tau(1 - \phi_2)}{1 + \tau} \right]^{\beta} (1 + \eta \phi_2)^{\beta} - (r + \delta) = 0.$$  

(14)
and $A_1$ and $A_2$ come from the homogenous solution and need to be determined by the boundary conditions. Mathematically, they represent the ‘diffusion part’ of the solution of the stochastic process.

As the wage becomes very high, the option value to hire approaches zero. As the wage becomes arbitrarily small, the option to fire becomes small. The hiring and firing options ($v^G_H$ and $v^G_F$) should therefore satisfy the boundary conditions $v^G_H(\infty) = 0$ and $v^G_F(0) = 0$: the firm will not consider the hiring (firing) option as the (stochastic) wage is high (low). Thus, we focus on the negative $\beta$ solution for $v^G_H$ and the positive $\beta$ solution for $v^G_F$. We write these conditions as

$$v^G_H(w) = A_2 [w(1 + \tau)]^{\beta_2}$$  \hspace{1cm} (15)

and

$$v^G_F(w) = A_1 [w(1 + \tau)]^{\beta_1}.$$  \hspace{1cm} (16)

We now add fixed marginal hiring ($H$) and firing ($F$) costs to the model with both $H$ and $F$ being payable by the firm.\(^{10}\) When there are fixed costs of either hiring or firing, the firm will consider the option value of maintaining her current position against the alternative of hiring or firing. In other words, it should be evident that the hiring and firing policy of the optimising firm is discontinuous. In some periods the optimal strategy of the firm will be to adjust the number of workers. Under other demand conditions a wait and see attitude will be chosen. More specifically, hiring and firing costs generate a corridor of inaction (status quo policy) within which firms do not change their workforce. This region is identified by the lower, $w_H$, and higher, $w_F$, control barriers. The definitions of the hiring and firing barriers, $w_H$ and $w_F$, are given by the value-matching and smooth-pasting conditions below. It is straightforward to show that according to the value-matching conditions the firm would find it optimal to exercise its option to hire or fire the marginal worker once $w$ hits one of the two barriers:

$$\frac{F(K) L^{1-\frac{1}{\sigma}}}{r + (1 - \sigma) \delta_\psi} - \frac{w_H}{w_H^{\beta_1}} \frac{w_H}{w_H^{\beta_2}}$$  \hspace{1cm} (17)

and

$$- \left[ \frac{F(K) L^{1-\frac{1}{\sigma}}}{r + (1 - \sigma) \delta_\psi} - \frac{w_F}{w_F^{\beta_1}} \frac{w_F}{w_F^{\beta_2}} \right]$$  \hspace{1cm} (18)

$^{10}$\(H\) can be thought of as representing the screening and training costs associated with the recruitment of a new employee and $F$ as the severance costs imposed by legislation when dismissing an employee.

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The left-hand sides of equations (17) and (18) show the marginal benefit from hiring/firing a worker and the right-hand sides the corresponding marginal costs. The marginal benefit of hiring a worker is equal to the sum of the present discounted value of his productivity net of wages and the value of the option to fire him. The firm’s ability to fire raises the benefit from employing a worker. The marginal cost of hiring is the sum of the direct hiring costs and the sacrificed option to hire him in the future. By hiring a worker today, the opportunity to do so in the future – when conditions may be more favourable – is sacrificed. Similarly, by firing a worker, the opportunity to do so in the future – when demand conditions may be even more adverse – is sacrificed, and the opportunity to hire is gained. The smooth-pasting conditions ensure that hiring (firing) is not optimal either before nor after the hiring (firing) threshold is reached. In technical terms, this means

\[
- \frac{1}{r + \delta + \eta(\lambda_1 \phi_1 - \lambda_2 \phi_2)} \tau
- \frac{1}{r + \delta - \lambda_1 \phi_1(1 - \eta - \eta \phi_1) + \lambda_2 \phi_2(1 - \eta + \eta \phi_2)}
+ \beta_1 A_1 (1 + \tau)^{\beta_1} w_H^{\beta_1 - 1} = \beta_2 A_2 (1 + \tau)^{\beta_2} w_H^{\beta_2 - 1}.
\]  

(19)

and

\[
\frac{1}{r + \delta + \eta(\lambda_1 \phi_1 - \lambda_2 \phi_2)} \tau
+ \frac{1}{r + \delta - \lambda_1 \phi_1(1 - \eta - \eta \phi_1) + \lambda_2 \phi_2(1 - \eta + \eta \phi_2)}
+ \beta_2 A_2 (1 + \tau)^{\beta_2} w_F^{\beta_2 - 1} = \beta_1 A_1 (1 + \tau)^{\beta_1} w_F^{\beta_1 - 1}.
\]  

(20)

Equations (17)–(20) form a non-linear system of equations with four unknown parameters, \(w_H\), \(w_F\), \(A_1\), and \(A_2\), and can be solved numerically once the solutions for \(\beta_1\) and \(\beta_2\) are obtained from equation (14). In order to visualise our approach to employment determination, we next consider calibrations of the model. These make the model amenable to graphical analysis.

III. Calibration and Results

The preceding section has laid out the model economy. Having illustrated that the stochastic framework has important ramifications for the dynamic behaviour of labour demand, we proceed in this section to use the theoretical models derived above to carry out a number of simulations to shed light on the workings of the models and the economic forces at work. For this reason, the model is calibrated in order to match characteristics of the German economy. In other words, an intuitive interpretation of the model is provided, and

\[11\] The numerical boundary value problem is solved with the method of Newton–Raphson for nonlinear systems. For a description of the algorithm used to compute the numerical simulations, see Press et al. (2002).

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throughout the remainder of the paper no background in stochastic calculus is necessary to understand the arguments in the text.

The unit time length corresponds to one year. Our base parameters are 

\[ \sigma = 0.05, \quad \lambda_1 = 0.1, \quad \lambda_2 = 0.1, \quad \phi_1 = 0.1, \quad \phi_2 = 0.1, \quad K = 1, \quad \delta = 0.1, \quad L = 1, \quad r = 0.05, \quad H = 0.1, \quad F = 0.6, \quad \psi = 1.5, \quad \eta = 0.1, \quad \mu = 0.4825, \quad \alpha = 0.3, \quad Z = 2.417, \quad \text{and the initial value of } \tau = 0.75. \]

Where possible, parameter values are drawn from empirical labour studies. The firing and hiring parameters are consistent with those in Bentolila and Bertola (1990) for Germany. Their estimated firing costs for Germany are in the range \(0.562 \leq F \leq 0.750\) and their hiring cost estimate (excluding on-the-job-training) for Germany is 0.066 of the average annual wage. Our specification \((H = 0.10)\) is also broadly consistent with the recruiting and training cost of two months in Mortenson and Pissarides’ (1999) calibration.\(^{12}\) They suggest that this number is consistent with survey results reported in Hamermesh (1993). Point estimates for \(\tau\) have been derived from Table 1. Finally, the price elasticity of demand parameter is set at \(\psi = 1.50\) as in Bovenberg et al. (1998). The determination of some parameters, however, requires the use of judgement, i.e. they reflect a back-of-the-envelope calculation.\(^{13}\)

To motivate the analysis of policy uncertainty, special attention has to be paid to the calibration of the Poisson processes. The Poisson process implies that the likelihood of a policy change is determined by the arrival rate \(\lambda\). This means that the time \(t\) one has to wait for the switch event to occur is a random variable whose distribution is exponential with parameter \(\lambda\):

\[
F(t) \equiv \text{prob}\{\text{event occurs before } t\} = 1 - e^{-\lambda t}. \tag{21}
\]

The corresponding probability density is

\[
f(t) \equiv F'(t) = \lambda e^{-\lambda t}. \tag{22}
\]

In other words, the probability that the event will occur sometime within the short interval between \(t_0\) and \(t_0 + dt\) is approximately \(\lambda e^{-\lambda t} dt\). In particular, the probability that it will occur within \(dt\) from now (when \(t = 0\)) is approximately \(\lambda dt\). In this sense \(\lambda\) is the probability per unit of time. Moreover, the number of policy changes \((x)\) that will take place over any interval of length \(\Delta\) is distributed

\(^{12}\) The OECD (1999) has compiled a comprehensive dataset describing legislative firing (procedural requirements, notification periods, severance pay, special requirements for collective dismissals, and short-time work schemes) and hiring costs (rules favouring disadvantaged groups, conditions for temporary and fixed-term contracts, training requirements) covering 22 indicators for 27 countries. These 22 indicators provide the inputs for the construction of cardinal summary indicators of employment protection across countries. These indicators of strictness of employment protection in the late 1990s are also available in the DICE database (see www.cesifo.de).

\(^{13}\) The calibrated model is not based on detailed time series data in the way econometric models are and does not have the predictive power of the latter. Note, however, that the goal of this paper is not to derive precise quantitative estimates of the impact of various labour market regulations, but rather to illustrate the qualitative predictions of the model, and to see what we can learn from this model.
according to the Poisson distribution

\[
g(x) \equiv \text{prob}\{x \text{ event occur}\} = \frac{(\lambda \Delta)^x e^{-\lambda \Delta}}{x!}
\]  

(23)

whose expected value is the arrival rate times the length of the interval \(\lambda \Delta\). We can back out from equation (23) the agent’s beliefs about policy changes. As a guide to calibration, Table 3 provides the probabilities that either one \((x = 1)\) or three \((x = 3)\) jumps will occur within 5 years \((\Delta = 5)\) or 10 years \((\Delta = 10)\) for the four arrival rates \(\lambda = 0.01, 0.05, 0.10\) and 0.15, respectively. For example, for \(\lambda = 0.05\) the probability that one jump will occur within 5 years is 19.5%.

In the simulations, a sensitivity analysis is performed over the grid \(\lambda_i \in \{0.0, 0.2\}\) for \(i = 1, 2\). First, we consider the employment thresholds for alternative hiring and firing costs. Despite the fact that deregulation of labour markets has ranked highly in European policy debates, few effective changes to the stringent nature of the employment constraints facing European firms appear to have been implemented over the last decade. Moreover, in a number of European countries the general trend towards greater employment protection would actually appear to have continued. The numerical results are given in Figure 1.

The major result of the calibrations is that higher hiring and firing costs lead to an increase of the no action area, i.e. increasing hiring and/or firing reduces the (lower) hiring threshold \((w_H)\) and increases the (upper) firing threshold \((w_F)\). On the one hand, laws designed to protect employees against firing dampen...
unemployment because existing workers are fired less easily. On the other hand, firing costs make it difficult for firms to fire workers, so firms hesitate to hire them in the first place, strengthening the hand of workers who already have a job. Fewer workers become unemployed, but those unlucky few are also less likely to find a job. In common with other studies [see, e.g. Bentolila and Bertola (1990)] our results indicate that an increase in firing costs has an asymmetric impact on the upper (firing) and lower (hiring) thresholds. The comparison of the left and right panel of Figure 1 indicates that higher firing costs discourage firing by more than it does hiring. Adjustment costs therefore do not necessarily imply a higher equilibrium rate of unemployment in this ‘shock-based’ story. But when firms are hit by a negative shock, laid-off workers have a more difficult time finding new jobs in a highly regulated labour market.

Figure 2 investigates numerically the impact of higher non-wage labour costs levels. As expected, a higher share of non-wage labour costs ($\tau$) leads to a decrease of $w_H$ and $w_F$. In particular, the downward shift of the hiring and firing trigger schedules implies that wage restraint is required to induce additional hirings.

Figure 3 provides a sensitivity analysis of the thresholds with respect to $\lambda_1$ and $\lambda_2$, i.e. we illustrate the impact of uncertainty about future non-wage labour costs upon the optimal hiring and firing thresholds. Alternatively, one may say that we consider different degrees of ‘policy-jumpiness’.

The 3-D graphs clearly indicate the entire no-action areas for alternative probabilities that non-wage costs ($\tau$) increase or decrease in the future. As is

\footnote{Note that the results are driven by the fact that firms perceive there to be the possibility of discrete changes in non-wage labour costs and is not therefore contingent on the changes being realised.}
evident from Figure 3, an increase in $\lambda_1$ leads to a downward shift of the hiring incentive (lower control barrier $w_H$), but has a small impact on the firing threshold $w_F$. On the contrary, an increase in $\lambda_2$ leads to an upward shift of the firing threshold (upper control barrier $w_F$). Resulting therefore, is a noticeable increase in the width of the no action corridor. The implication is that the interaction between partial irreversibility and a more volatile environment characterised by ‘policy-jumpiness’ can have important ramifications for employment dynamics.

Let us now consider changes in $\sigma$. In other words, we analyse the sensitivity of the optimal thresholds with respect to changes in the volatility of the geometric Brownian motion representing direct wage (input price) uncertainty. The simulation results in Figure 4 indicate that the threshold value at which hiring takes place is increasing in the ‘noisiness’ level even though the firm is risk neutral. In volatile environments, the best tactic is to keep options open and await new information rather than take an employment decision today. The intuition is that the firm can counteract the impact from additional uncertainty by a wait and see attitude for the time being. The policy implication is that countries with explicit or implicit long-term wage accords in place reducing wage uncertainty will achieve a better employment performance.  

Figure 5 provides a sensitivity analysis of the thresholds with respect to $\eta$. In other words, we are considering different degrees of shifting of the burden of payroll taxes from employers to workers. The results indicate that the no

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15 There are various examples of successful employment pacts at national level. Long-term wage accords in several European countries typically had several common features. First, they were agreements between trade unions and employer associations. Second, they were agreements over a several year period. Most of the agreements contained elements of wage moderation, with a vague prospect of higher employment and higher economic growth.

16 Empirical evidence on the impact of payroll taxation upon wages for different countries is mixed and remains highly controversial. Results range from little shifting to full-shifting [see, e.g. Kugler and Kugler (2003)].
action area is narrowing slowly, but the nature of the results is not particularly sensitive to this parameter value.\textsuperscript{17}

The traditional literature has often focused exclusively on the impact of certain labour market regulations, largely ignoring the role of product market regulations and the interactions between regulatory interventions in the two markets. In recent years, however, an increasing literature analysing such interactions has emerged (see, e.g. the careful discussion in Blanchard and Giavazzi, 2003). As a step ahead in the analysis, we therefore provide an initial attempt to quantify such interactions. Policies in product and labour markets are normally aimed at influencing outcomes in the markets to which they directly apply. However, the empirical and theoretical findings of various recent papers suggest that policy interactions between product and labour markets can have important effects, sometimes even having an impact comparable to within-market policy influences. For instance, in some European countries, anti-competitive product market regulations and strict employment protection policies appear to have contributed equally to keeping employment rates low (see Nickell, 1999; Fonseca \textit{et al.}, 2001; Nicoletti \textit{et al.}, 2001; Bertrand and Kramarz, 2002).\textsuperscript{18} How do alternative patterns of regulations in product markets influence the hiring and firing decisions of firms in our modelling framework? In the simulations below, we think of the regulatory stance on the

\textsuperscript{17}The mathematical explanation is simple. The ‘symmetrical’ baseline parameters values $\phi_1 = \phi_2 = 0.10$ ensure that the effect of $\eta$ on the particular solutions is zero. We will come back to this assumption below.

\textsuperscript{18}A set of indicators of various dimensions of product market regulation shedding some light on cross-country differences is available at http://www.cesifo.de/pls/diceguest/search.create_simple_search_page.

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product markets as being captured, admittedly in abstract fashion, by the degree of product market competition, $\psi$. The aim of the simulations is to assess the policy relevance of cross-market effects from product markets to labour markets.

Figure 6 plots the employment thresholds as a function of $\psi$. As expected, anticompetitive product market regulations are found to increase the employment thresholds and to widen the inaction range. The cross-market interaction results therefore suggest that strict product market regulation is likely to affect employment negatively. Despite its preliminary character, the analysis thus suggests that the removal of barriers to competition in potentially competitive markets can be a complement to labour market reforms.\footnote{There are at least three channels through which the strictness of product market regulations may have implications for labour markets. (a) Product market deregulation increases competitive pressures among firms, raising the elasticity of product demand. At the firm level, for given wages, a higher demand elasticity raises labour demand; (b) product market deregulation lowers entry costs. This is likely to lead to higher employment; (c) a more competitive institutional setting will also contribute to a more innovative and dynamic economy through thriving entrepreneurial activity (Acemoglu \textit{et al.}, 2002). While the intensity of these effects will depend also on the features of labour market institutions, their sign will generally remain the same across different institutional settings. This leads to the ‘all or nothing’ warning issued by Coe and Snower (1997) and Orszag and Snower (1998). They argue that piecemeal labour market reforms may have had so little success because they disregarded the complementarities between a broad range of policies and institutions.}

Since the focus of the paper is employment, we next present a translation from thresholds to employment and assess the impact of policy-jumpiness upon $L$. In order to get a clear ‘feel’ for the dynamics of the model, we first have to specify a solution method that will lead us to generate discrete realisations of the level of employment, given the chosen levels of parameters. Several options are

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{The threshold values as function of $\eta$.}
\end{figure}
available at this point, but the structure of the model readily suggests using a sequential iterations method. It works as follows. Equation (4) is proxied by the following discrete stochastic differential equation – the Euler scheme,

\[ w_{t+\Delta t} - w_t = \sigma w_t \varepsilon_t \sqrt{\Delta t} + \eta \frac{w_t \Delta \tau_t}{\tau_t}, \quad \varepsilon_t \sim N(0, 1) \quad \text{for } i = 0, 1, \tag{24} \]

where the normal random variables, \( \varepsilon_t \), are generated via the central limit theorem and the Box–Muller (1958) method for transforming a uniformly distributed random variables to a normal distribution with given mean and variance, and \( \Delta \tau_t \) represents two jump processes of equation (6) proxied by two separate Poisson random generators. Superimposed on the graphs are the resulting \( \tau \) series.

As the time passes, the term \( w_t \) fluctuates according to the corresponding stochastic processes and \( L \) will depreciate as long as \( w_t \) is staying within the no-action area. If \( w_t \) hits the thresholds \( (w_H \text{ or } w_F) \), the firm will hire (or fire) employees to raise (or lower) \( L \) so that \( w_t \) is again within the no-action area. Three different sample paths of the stochastic adjustment process over 30 years are given in Figure 7.\(^{21}\) We immediately see that an increase or a decrease in employment always occurs when the firm ‘by accident’ hits the relevant threshold.\(^{22}\)

\(^{20}\) See, e.g. Press et al. (2002) for a description of the algorithm.

\(^{21}\) The initial value for \( w_t = 0 \) is 1.000 and \( \Delta t = 0.02 \). All other parameters are as in the benchmark case. It should be pointed out that Figure 7 displays the results for three different seeds for the random number generator, and might not represent a typical or average path.

\(^{22}\) We ignore behavioural assumptions regarding market rivalry, which in turn would necessitate some kind of game-theoretic analysis to take account of the strategic interactions
All the above discussion has been concerned with the employment decision of a single firm. From a macroeconomic point of view, however, the question of primary interest is the impact of uncertainty and (partial) irreversibility on aggregate employment. Yet it is obvious that one cannot just translate mechanically the above microeconomic partial equilibrium results to aggregate employment. To assess the role of irreversibility in aggregate factor demand it is essential to take explicitly into consideration the heterogeneity of individual firms’ hiring decisions. Furthermore, attention has to be paid to the distinction between idiosyncratic and aggregate shocks, and to potentially contrasting implications of these shocks to the dynamics of aggregate employment. Suppose that we re-interpret the model at the macroeconomic level, i.e. $L$ now represents aggregate employment. Unlike microeconomic data, aggregate employment series look smoother since microeconomic adjustments are far from being perfectly synchronised. The question arises as to whether aggregation eliminates all traces of infrequent lumpy microeconomic adjustment. We again focus on employment ($L$), and we model aggregate employment in terms of average employment of a number of individual firms indexed by $i \in [1, 200]$. Experimentation with larger numbers of runs ($i$) shows no significant change to the results. Solving the model in this fashion, gives the aggregate employment data in Figures 8 and 9. In order to demonstrate the impact of non-wage labour cost uncertainty, we consider two alternative scenarios. In Figure 8, we assume a ‘symmetric scenario’ where the probability of an increase in non-wage labour costs in the future ($\lambda_1$) is identical to the corresponding probability of a fall in future payroll taxes ($\lambda_2$). On the contrary, in Figure 9 we simulate an ‘asymmetric scenario’ with $\lambda_1 > \lambda_2$, i.e. we assume a (slightly) higher probability of higher non-wage costs in the future. To make the Figures easier to interpret, we have also superimposed the employment series excluding policy jumpiness ($\lambda_1 = \lambda_2 = 0.0$).

The graphs suggest three insights related to the evolution of employment $L$. First, ‘policy-jumpiness’ leads to medium-run swings in employment, reflecting the joint impact of both Poisson processes to employment. Second, policy-jumpiness has no long-run impact upon employment in the ‘symmetric case’ ($\lambda_1 = \lambda_2$), i.e. lower employment is a medium-term phenomenon and both series converge to the same employment in steady state. Third, in the slight asymmetric scenario both series don’t converge to the same employment in steady state.

This aspect of the simulation is a desirable outcome which is supported by evidence in Caballero et al. (1995). The authors (1995) propose several among the firms, results of which are in turn heavily dependent on assumptions regarding the information sets available and the type of game being played. The ramifications of competitive interaction on the decision making of firms have been discussed by Smit and Ankum (1993) and Leahy (1993). Leahy (1993) has shown that the assumption of myopic firms who ignore the impact of other firms’ actions results in the same critical boundaries that trigger factor demand as a model in which firms correctly anticipate the strategies of other firms. Grenadier (2002) has recently extended Leahy’s (1993) ‘Principle of Optimality of Myopic Behavior’ to the apparently more complex case of dynamic oligopoly under uncertainty. Both papers therefore permit to bypass strategic general equilibrium considerations when analysing factor demand under uncertainty.

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Figure 7. Three sample paths of the wage ($w$), the hiring and firing thresholds ($w_H$ and $w_F$), and optimal employment ($L$).
frameworks in which the probability distributions of the state of individual agents are formally linked to cross-sectional distributions at aggregate levels. The paper suggests that idiosyncratic shocks tend to smooth out microeconomic rigidities by spreading agents in state space, while aggregate policy-induced shocks tend to coordinate individual agents’ actions, thereby allowing hysteretic
behaviour at the microeconomic level to influence the dynamics of aggregate time series.

IV. Summary Remarks and Conclusions

Although one has to be careful when drawing conclusions from this type of modelling framework, some interesting issues emerge from this simulation study. First, it appears that focusing on the level of hiring and firing costs \textit{per se} is inappropriate because adjustment costs have considerably more of an impact, the higher the prevailing degree of uncertainty. In other words, the interaction of labour market institutions with changes in the economic environment is the most plausible candidate for explaining rising unemployment.\footnote{See Minford and Naraidoo (2002). This explanation is in fact also the gist of papers by Blanchard and Wolfers (2000), Chen \textit{et al.} (2002) and Ljungqvist and Sargent (2002).} Economic conditions have become more volatile over the last ten years due to globalisation and the transition to the new economy, while at the same time labour market institutions have by and large been kept unchanged over the last thirty years in the unholy triple alliance of reform laggards (France, Germany and Italy).\footnote{Recently, however, chancellor Schröder and his government have implemented their \textit{Agenda 2010}. They have ratified substantial cuts in the duration and amounts of unemployment and non-wage labour costs. The government has also weakened job-protection in small companies to encourage employers to hire new workers. Our exercise clearly indicates that such a reform package – if boldly and fully implemented – will push the German economy in the right direction.} Second, our results imply that steps towards labour market liberalisation have considerably more of an impact on employment in countries with long-term wage accords and wage restraint, such as the Netherlands. The hope is that some of the insights gained from this exercise shed light on the determinants of employment and will carry over into the next round of analysis.

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Appendix A: Derivation of Equation (11)

Assume that the particular solution for the shadow price of employees has the following functional form as the particular integral components:

\[ v = BF(K)L^{\frac{1}{\psi}} + Cw + Dw, \tag{A1} \]

where \( B, C, \) and \( D \) are unknown constants. Then, we have

\[ v_{\text{ww}} = 0, \tag{A2} \]

\[ -\delta L^\psi v_L = -\delta BF(K)\left(\frac{1 - \zeta}{\psi} - 1\right)L^{\frac{1}{\psi} - 1}, \tag{A3} \]

\( \psi \)
\[ \lambda_1 [v(w(1-\eta \phi_1), [1 + \tau (1 + \phi_1)]) - v] \]
\[ = \lambda_1 [Cw(1-\eta \phi_1) + Dw(1-\eta \phi_1)\tau (1 + \phi_1) - Cw - Dw\tau] \]
\[ = -\eta \lambda_1 \phi_1 Cw + \lambda_1 \phi_1 (1-\eta \phi_1) Dw\tau, \] (A4)
\[ \lambda_2 [v(w(1+\eta \phi_2), [1 + \tau (1 - \phi_2)]) - v] \]
\[ = \lambda_2 [Cw(1+\eta \phi_2) + Dw(1+\eta \phi_2)\tau (1 - \phi_2) - Cw - Dw\tau] \]
\[ = +\eta \lambda_2 \phi_2 Cw - \lambda_2 \phi_2 (1-\eta + \eta \phi_2) Dw\tau. \] (A5)

Substituting into equation (8) in the text yields
\[ \left[ \left( r + \delta \frac{1 - \alpha}{\psi} \right) B - 1 \right] F(K) \frac{L^{\frac{\alpha}{\psi}} - 1}{K^{\frac{\alpha}{\psi}}} + [(r + \delta + \eta(\lambda_1 \phi_1 - \lambda_2 \phi_2)) C + 1]w \]
\[ + [(r + \delta - \lambda_1 \phi_1 (1 - \eta - \eta \phi_1) + \lambda_2 \phi_2 (1 - \eta + \eta \phi_2)) D + 1] w\tau = 0. \] (A6)

Equation (A8) must hold for any value of \( B, C, \) and \( D, \) so that
\[ B = \frac{1}{r + \delta \frac{1 - \alpha}{\psi}}, \] (A7)
\[ C = \frac{-1}{r + \delta + \eta(\lambda_1 \phi_1 - \lambda_2 \phi_2)}, \] (A8)
\[ D = \frac{-1}{r + \delta - \lambda_1 \phi_1 (1 - \eta - \eta \phi_1) + \lambda_2 \phi_2 (1 - \eta + \eta \phi_2)}. \] (A9)

It is then straightforward to obtain equation (11).

**Appendix B: Derivation of Equations (13) and (14)**

The homogeneous solutions to equation (12) should have the same components as the particular solutions with respect to the uncertainty variables. Assume the homogeneous solutions have the functional form
\[ v = A[w(1 + \tau)]^\beta. \] (B1)

Then we have
\[ \frac{1}{2} \sigma^2 w^2 v_{ww} = \frac{1}{2} \sigma^2 \beta (\beta - 1) A[w(1 + \tau)]^\beta, \] (B2)
\[ \lambda_1 [v(w(1-\eta \phi_1), (1 + \tau (1 + \phi_1))) - v(w, 1 + \tau)] \]
\[ = \lambda_1 A[w(1-\eta \phi_1)]^\beta [(1 + \tau (1 + \phi_1))]^\beta - (1 + \tau)^\beta, \] (B3)
\[ \lambda_2 [v(w(1+\eta \phi_2), (1 + \tau (1 - \phi_2))) - v(w, 1 + \tau)] \]
\[ = \lambda_2 A[w(1+\eta \phi_2)]^\beta [(1 + \tau (1 - \phi_2))]^\beta - (1 + \tau)^\beta. \] (B4)
Now substitute into equation (12) in the text.

\[
\begin{align*}
\left\{ \sigma^2 \beta (\beta - 1)(1 + \tau)^{\beta}/2 + \lambda_1 [(1 + \tau (1 + \phi_1))^\beta \\
-(1 + \tau)^\beta] (1 - \eta \phi_1)^\beta + \lambda_2 [(1 + \tau (1 - \phi_2))^\beta \\
-(1 + \tau)^\beta] (1 + \eta \phi_2)^\beta - (r + \delta) (1 + \tau)^\beta \right\} A^\beta = 0.
\end{align*}
\] (B5)

It is straightforward to obtain the following characteristic equation since the bracketed item must equal zero:

\[
\frac{1}{2} \sigma^2 \beta (\beta - 1) + \lambda_1 \left[ \left( \frac{1 + \tau (1 + \phi_1)}{1 + \tau} \right)^\beta - 1 \right] (1 - \eta \phi_1)^\beta \\
+ \lambda_2 \left[ \left( \frac{1 + \tau (1 - \phi_2)}{1 + \tau} \right)^\beta - 1 \right] (1 + \eta \phi_2)^\beta - (r + \delta) = 0.
\] (B6)

Note that there are two roots for characteristic equation (B6). Therefore, the general solutions are denoted by

\[ v^G = A_1 [w(1 + \tau)]^{\beta_1} + A_2 [w(1 + \tau)]^{\beta_2}, \] (B7)

where \( \beta_1 > 0 \) and \( \beta_2 < 0 \).

References


