Stock market wealth effects in an estimated DSGE model for Hong Kong

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1. Introduction

The combination of dynamic stochastic general equilibrium (DSGE) techniques used in the real business cycle literature with the Keynesian rigid prices assumption lies at the heart of the recent growth in New Keynesian models of macroeconomic analysis. DSGE models address flaws in the standard Keynesian model and, as equilibrium business cycle theorists like to point out, the accompanying world-view. Moreover, they allow for easy-to-handle small- and mid-scale models with excellent predictive properties by log-linearizing the first-order conditions of optimizing households and firms. For an underlying sticky price framework, these models reveal monetary policy to have real effect over the short-run. Finally, they cannot be criticized like conventional aggregate demand-aggregate supply models for lacking detailed microfoundations.

DSGE models are also useful in macroeconomic forecasting. For example, Del Negro et al. (2007) apply DSGE models to constrain VARs in a Bayesian setting. Theoretical restrictions from firms and households optimizing behavior subject to their intertemporal budget constraints imply a set of cross-equation restrictions on the VAR parameters. Del Negro et al. (2007) relate these constraints to priors on the model parameters (informative priors pull the parameters toward the theoretical constraints, while ignoring theory tends to diffuse priors). In their simulation study, they found evidence that a DSGE-based VAR yields better out-of-sample forecasting performance for a range of macroeconomic variables than the usual unconstrained VAR.

A vast body of studies seeking to capture Hong Kong’s macroeconomic relationships through various ad hoc and partial equilibrium models has been generated in recent years. All these studies, however, omit explicit microfoundations, including the now-standard assessment of optimizing behavior of rational agents. Our contribution here is an attempt to apply a New Keynesian approach with adjustments to capture Hong Kong’s unique macroeconomic features. Economic transformation has turned Hong Kong into a global financial center with a massive concentration of financial and business services. Much like a city-state, Hong Kong’s economic structure resembles a high-income metropolis rather than a typical advanced economy. Its growing integration with Mainland China tracks developments in the adjacent manufacturing-centered Pearl River Delta with which it shares strong trade and service linkages.2

Given the relatively stable inflation rates of the last two decades and the rise of financial markets, the latest developments in New Keynesian modeling focus on the interaction of monetary policy and

1 The growing body of literature indicates that many researchers have found equilibrium business cycle theory worthy of formal analysis. See Clarida et al. (1999) or Goodfriend and King (1997) for surveys. Paul Krugman dismisses the value of this work, noting recently that “economists who have spent their entire careers on equilibrium business cycle theory are now discovering, in effect, that they invested their savings with Bernie Madoff.” http://www.krugman.blogs.nytimes.com/2009/01/27/madoff-economics/

2 For a discussion of the role of global cities in the globalized production process, see Glaeser et al. (2001).

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financial stability, and specifically, the question of whether central bankers should respond to stock market dynamics. By enhancing the ideas of Yaari (1965) and Blanchard (1985), a new strand of investigation has been developed that gives the possibility to include short-run wealth effects that consider stock-price dynamics in the optimal intertemporal consumption decision (see e.g. Nisticò, 2005; Di Giorgio and Nisticò, 2007). Firms issue equity shares to the public (based on the tree model of Lucas, 1978) and households face a constant probability of economic death. The expected lifetime of a household (creation to dissolution) is equal to the time horizon relevant for decision-making.

A well-known drawback of the standard infinite-lifetime approach is that it neglects any effects of wealth upon consumption, since aggregated consumption is simply described by an Euler equation, linking today’s consumption to the expected discounted future value. Contrary to this, we adopt a perpetual-youth approach, were new born cohorts with zero assets interact with old cohorts on financial markets. Since newcomers haven’t been able to accumulate wealth with which they could smooth their consumption profile, the average expected level of consumption will be lower, when they enter the market and replace some of the old agents. Intuitively speaking, today’s agents anticipate any increase in stock prices by increasing their individual consumption, but a fraction of these agents will be replaced by newcomers with zero assets, whose consumption pattern is unaffected, as they haven’t been in the market at the time of the increase. Consequently, current average consumption is affected stronger than expected future average consumption. This introduces a wedge between the stochastic discount factor and the average marginal rate of intertemporal substitution in consumption and makes stock-price dynamics relevant for aggregate consumption. As numerous recent empirical studies have shown such effects to be important for the Hong Kong economy, we allow for them using this approach.

The remainder of this paper is organized as follows. Section 2 briefly presents stylized facts on the Hong Kong economy. Section 3 describes the model, reflecting these salient features. Estimation and inference of the DSGE model are laid out in Section 4. We conclude with a summary and evaluation of our findings.

2. Structural characteristics of Hong Kong

Hong Kong’s currency board stands out among such arrangements around the world as one with the longest history. Hong Kong’s currency board was established in 1983 and the Hong Kong dollar was pegged to the US dollar at 7.80 to 1. In September 1998, the rate was changed to 7.75 to 1. The exchange rate moved gradually from 7.75 back to 7.80 between April 1999 and August 2000. The HKD is freely convertible and the Hong Kong Monetary Authority (HKMA) is responsible for the peg. Given the currency board system, Hong Kong’s interest rates are largely determined exogenously by their US-dollar counterpart adjusted by a risk premium. Property and equity prices are important factors in determining household wealth, and thus, domestic demand. It is our expectation here that stock market wealth effects are significant in the case of Hong Kong, so we concentrate on understanding the transmission of stock-price dynamics to consumption. This study distinguishes between effects from asset holdings and those from real estate, and focuses solely on the former. Analysis on the influence of housing prices is left to future research.

A number of papers investigate the relationship between consumption income and wealth in Hong Kong. By estimating an IS curve relationship, including the Hang Seng index and a property price index, Ha et al. (2002) find a significant elasticity of output to share prices of 0.04. The estimates of Cutler (2004) indicate a long-run financial wealth elasticity of 0.085 by estimating a consumption function using notes and coin held by the public, bank deposits, and the market capitalization of the Hang Seng index as a proxy for gross financial wealth. This value is comparable with his findings for the U.K. (0.088) and France (0.101), but smaller than in the US (0.23). The study of Liu et al. (2007a) gives additional support for a financial wealth effect using different VAR specifications. For a sample period of 1984Q1–2006Q4 an unanticipated increase in financial wealth (derived from stock prices and market capitalization) is estimated to be between 0.039 and 0.046, while for a subsample from 1997Q3 to 2003Q1 the estimations increase from 0.101 to 0.122. On the other hand, much smaller estimates are found by Liu et al. (2007b), who use an instrumental variable approach to estimate a habit formation model of consumption and find a short-run marginal propensity to consume from financial wealth of only 0.0016, which is compatible with a long-run value of 0.007.

Neoclassical and New Keynesian economists disagree as to how quickly wages and prices adjust. Neoclassical economists build their macroeconomic theories on the assumption that wages and prices are flexible. Prices clear markets, i.e. balance supply and demand, by adjusting quickly. New Keynesian economists, in contrast, believe that market-clearing models cannot explain short-run economic fluctuations. They advocate models with sticky wages and prices. A reason prices do not adjust immediately to clear markets is that adjusting prices is costly.

Furthermore, the adjustment of prices throughout the economy is staggered. Yetman (2009) provides a recent analysis of the flexibility of prices in Hong Kong. In broad terms, the empirical results indicate that prices of consumer goods and services in the Hong Kong CPI are comparatively flexible. Since Hong Kong’s currency board system precludes a systematic policy response to domestic shocks, adjustment has to occur largely through price flexibility. In the estimates below, we treat the degree of persistence as an unknown parameter to be estimated.

Since all of these studies omit explicit microfoundations, our contribution to this literature is the assessment of a New Keynesian DSGE model designed to capture many of Hong Kong’s unique and salient features—a currency board system, a resultant high sensitivity of foreign monetary policy, economic structure as a financial center, and comparatively flexible prices.

---

5 Studies providing a rationale for a reaction to stock-price misalignments include Cecchetti et al. (2000, 2002) and Cecchetti (2003), whereas Bernanke and Gertler (1999, 2001) argue that the only desirable reaction to stock prices is the one implicit in the response to expected inflation and output.

4 Limited planning horizons (or impatience) in economic decision-making can be due to disconnection of current households from future generations or the lack of an altruistic bequest motive. They can also occur through imperfect access to financial markets. Recent work has relaxed the assumption of unrestricted financial-market access. In Lechtko (2005), for example, access to international financial markets is restricted to certain households as the paper is focused on explaining the relatively high volatility of consumption in emerging markets.

5 Liu et al. (2007a) find evidence for both anticipated and unanticipated income and wealth effects to influence Hong Kong’s consumption, especially after the 1997 financial crisis.

6 Under a pure currency board system, the Hong Kong dollar and the US dollar interest rates should move in tandem. Several factors, however, including the expectation that the Hong Kong dollar should also be allowed to reflect renminbi appreciation, has caused the two rates to deviate slightly.

7 An estimated DSGE model that includes the housing market can be found in Iacoviello and Neri (2010). While this is certainly an issue worth pursuing eventually, dealing with the housing market here would require modeling of an explicit housing sector in addition to the conventional consumption sector and an assumption that each household supplies labor to both sectors. Thus, we focus on stock prices and do not investigate the effects of Hong Kong’s real estate prices. To be fair, one will scarcely find a business cycle model that is able to mimic all features of an economy—that is why it is a model.

8 New Keynesian macroeconomics (Mankiw, 1985; Ball and Romer, 1989) often claims that menu costs can have large costs for society. Expansions are welfare-improving as they bring the economy closer to the social optimum; recessions are welfare-reducing as they push the economy further from the social optimum. Does this imply that business cycles are costly on average? It is noteworthy that Call et al. (2007) have shown that menu costs don’t generate sizable welfare costs.
3. The model

The DSGE model we develop in this paper allows for the impact of wealth effects on an agent’s consumption pattern. The framework is based primarily on the closed economy framework with wealth effects of Nisticò (2005) and the seminal work of Galí and Monacelli (2005). The study of Lubik and Schorfheide (2007) also serves as a useful empirical benchmark.9 In addition to this, Justiniano and Preston (2010) have estimated a small open-economy DSGE model for Australia, New Zealand, and Canada. However, their focus is on optimal monetary policy design and they consider the case of flexible exchange rates. In order to capture the specific characteristics of the Hong Kong economy, and specifically, the influence of stock prices on consumption, the conventional open-economy DSGE framework is reformulated so that the Euler equation for consumption captures the impact of stock-price dynamics. As we deform the domestic economy as one among a continuum of infinitesimally small economies, it is of negligible size relative to the rest of the world.10 The resulting model depends on specific structural parameters such as the openness of the domestic economy, the substitutability of goods of different economies, and the probability that an agent’s consumption will cease during a given period due to unforeseen circumstances, such as death.

Since we focus on the interaction of one economy with the rest of the world, we consider this to be the domestic country and do not use an i-index for variables of this country. Instead, variables with an index i ∈ {0, 1} refer to one economy i among the continuum that represents the rest of the world. The superscript * here refers to all foreign countries (the rest of the world as a whole). In what follows, we describe the microfoundations of foreign countries only where we deem it necessary. Otherwise, the foreign country counterparts of domestic equations can be derived in a straightforward manner. If not explicitly mentioned, price indices are measured in domestic currency units. To help the reader follow the analytical derivations, Tables 1–3 define all variables and parameters used in this section.

The economy is inhabited by households and firms. Both face the conventional New Keynesian optimization problems based on a competitive goods market (drawn from Dixit and Stiglitz, 1977) and sticky price setup (drawn from Calvo, 1983). In two-stage production, final goods producers (retail sector) use the output of intermediate goods producers (wholesale sector) as input. Final goods producers are assumed to produce competitively using a CES technology consisting of a continuum of nontraded intermediate goods. We further assume that intermediate firms issue equity shares.11 Thus, households can hold two types of financial assets: state-contingent bonds and equity shares of intermediate firms. If nominal gross returns on internationally tradable state-contingent bonds are equalized across countries (international risk sharing), we can show that the uncovered interest parity (UIP) assumption holds. Furthermore, the Law of One Price (LOOP) holds at the brand level, implying purchasing power parity (PPP) at any point in time.12 We follow the empirical literature and allow for additional backward-looking elements by assuming that a fraction of backward-looking price-setters exists.

3.1. Households

3.1.1. Individual optimization

3.1.1.1. Intertemporal allocation. Following Blanchard (1985), each economy consists of an indefinite number of cohorts facing a constant probability of dying each period. The implied expected lifetime of 1/y can be interpreted as the effective decision horizon of consumers. As in standard DSGE models, households demand consumption goods

<table>
<thead>
<tr>
<th>Table 1</th>
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<tbody>
<tr>
<td><strong>Financial market variables and prices.</strong></td>
</tr>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>Q_t^k</td>
</tr>
<tr>
<td>Q_t^l(Q_{t}^{*})</td>
</tr>
<tr>
<td>D_t</td>
</tr>
<tr>
<td>B_t^j,B_l^{j−k}</td>
</tr>
<tr>
<td>B_t^{j*k},B_l^{j−k}</td>
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<tr>
<td>D_t(k)</td>
</tr>
<tr>
<td>ζ_t(k)(ζ_{t}(k))</td>
</tr>
<tr>
<td>Ω_t</td>
</tr>
<tr>
<td>F_t^{i−1}</td>
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<tr>
<td>μ_t(k_i)</td>
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<tr>
<td>W_t</td>
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<tr>
<td>P_t^i(P^*_t)</td>
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<td>P_t^i(I_t)</td>
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<td>P_t^i(P_t^*l)</td>
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<td>P_t^i(P_t^*k)</td>
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<tr>
<td>P_t^i(k)(P_t^*l(k))</td>
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<tr>
<td>P_t^i(l)(P_t^*l(k))</td>
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<tr>
<td>β_t</td>
</tr>
<tr>
<td>p_t^i(p_t^*l(k)),p_t^i(p_t^*l(k))</td>
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<tr>
<td>P_t^l, P_t^*l</td>
</tr>
<tr>
<td>H_t</td>
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<tr>
<td>n_t</td>
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<tr>
<td>n_t(π_t)</td>
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<tr>
<td>S_t^k(S_t^l)</td>
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<thead>
<tr>
<th>Table 2</th>
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<tbody>
<tr>
<td><strong>Household variables and additional indices.</strong></td>
</tr>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>B_t^{i}</td>
</tr>
<tr>
<td>B_t^{i,j}</td>
</tr>
<tr>
<td>B_t^{i,j}</td>
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<tr>
<td>G_t(P_t^i)</td>
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<tr>
<td>C_t^i(G_t^i)</td>
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<tr>
<td>C_t^i(C_t^j)</td>
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<tr>
<td>C_t^i(C_t^j)</td>
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<tr>
<td>C_t^i(k)(C_t^j(k))</td>
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<td>C_t^i(k)(C_t^j(k))</td>
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<tr>
<td>N_t^i</td>
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<tr>
<td>N_t^l</td>
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<tr>
<td>Y_t^i(Y_t^j)</td>
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<tr>
<td>Y_t^l</td>
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<tr>
<td>X_t(k)</td>
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<tr>
<td>Δ_t</td>
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<td>T_t</td>
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<td>ρ_t</td>
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<td>M_{t}</td>
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<td>W_t</td>
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<tr>
<td>H_t</td>
</tr>
<tr>
<td>A_t</td>
</tr>
<tr>
<td>Δ_t+1</td>
</tr>
</tbody>
</table>

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9 Although Lubik and Schorfheide (2007) estimate small open economy DSGE models for four countries, their focus is on whether or not central banks in the economies included in the study respond to exchange-rate movements.

10 Since each individual economy is represented in the model by a value of zero, its policy decisions have no impact on the rest of the world. For convenience, we assume identical preferences, technology, and market structure.

11 The assumption that only intermediate firms issue equity shares is made purely for convenience. The intuition behind is that the shares of final goods producers will be summarized by one single share, since the retail sector is assumed to produce competitively, while firms in the wholesale sector are endowed with some monopolistic power and face price rigidity, resulting in a continuum of different firms with different equity shares on the [0, 1] interval. Adding one share for the final goods sector would not change the resulting model, but complicate the derivations.

12 For simplicity, we assume that LOOP, and therefore PPP, holds. As shown in Lubik and Schorfheide (2005), deviations from PPP result in a Phillips curve relationship between import price inflation and the LOOP price gap.
and supply labor. Each cohort $j$ is allowed to buy two types of financial assets: domestic and foreign state-contingent bonds (which holdings of a cohort $j$ are denoted by $B_t^j$ for domestic bonds and $B_t^{j,F}$ for bonds from country $i$) and equity shares of domestic intermediate firms (denoted as subscript $k$), for which nominal prices are given by $Q_t^j(k)$.

Before describing the decision problems of households, we briefly clarify our notation. $Z_t(k,j)$ is defined as the equity shares of a firm $k$ held by the cohort $j$. The corresponding nominal composite dividend yield is given by $D_t^j(k)$. Following this notation, the nominal financial wealth of a domestic individual of cohort $j$ is given by

$$
\Omega_t(j) = \frac{1}{1-\gamma} \left[ B_t^j + \int_0^t \frac{1}{P_t} E_t \left[ \frac{1}{1-\gamma} \frac{B_{t+1}^{i,F}(j)}{P_{t+1}} + \int_0^1 \left( Q_t^j(k) + D_t^j(k) Z_t(k,j) \right) dk \right] \right].
$$

where $Z_t^i$ is the nominal bilateral exchange rate (defined as the domestic price of country $i$’s currency) and firms are normed to one. Following Eq. (1), financial wealth includes bond holdings of the previous period and the payoff on the portfolio of equity shares (dividends plus market value). Since financial wealth also pays off the gross return on the insurance contract, redistributing the financial wealth of those who died among the surviving consumers of a cohort $j$, total personal financial wealth is accrued by $D_t^j(k)$. Each cohort $j$ chooses its optimal consumption-leisure decision and Eq. (5) is the Euler equation, describing the optimal intertemporal consumption pattern (the equilibrium stochastic discount factor for one-period ahead nominal payoffs equals the time-discounted stochastic growth in the marginal utility of consumption). Furthermore, Eq. (6) is the optimal intertemporal decision concerning the holdings of equity shares and equates the nominal price of a portfolio to its nominal expected payoff one-period ahead, discounted by $F_{t+1}$. This defines the stock-price dynamics. The first-order conditions ensure that at utility maximum the representative household cannot benefit from feasible intertemporal consumption reallocations.

### 3.1.1.2. Intratemporal allocation
Consumers consume a composite that is a Dixit–Stiglitz aggregator over differentiated goods. Households may purchase domestic and foreign consumption goods. The composite consumption index of a domestic cohort $j$ is defined as

$$
C_t(j) \equiv \left[ \frac{1}{1-\gamma} \alpha C_{Hi,t}(j)^{\frac{\gamma-1}{\gamma}} \left[ \frac{1}{\alpha} C_{Hi,t}(j)^{\frac{\gamma-1}{\gamma}} + 1 \right] \right]^{\frac{\gamma}{\gamma-1}},
$$

where $C_{Hi,t}(j)$ and $C_{Hi,t}(j)$ represent the domestic demand for domestic and foreign final goods, respectively. $\alpha > 0$ represents the elasticity of substitution between domestic and foreign goods, and $\alpha$ refers to the share of domestic consumption allocated to imported goods and is thus a natural index of openness. Furthermore, the demand indices are given by the following CES functions

$$
C_{Hi,t}(j) \equiv \left[ \int_0^1 C_{Hi,t}(j)^{\frac{1}{\gamma}} \frac{1}{\gamma} \right]^{\frac{1}{1-\gamma}}, \quad C_{Hi,t}(j) \equiv \left[ \int_0^1 C_{Hi,t}(j)^{\frac{1}{\gamma}} \frac{1}{\gamma} dt \right]^{\frac{1}{1-\gamma}}.
$$

where $\epsilon$ denotes the elasticity of substitution between the differentiated goods within one single country and $\xi$ measures the substitutability between goods produced in different foreign countries. $C_{Hi,t}(j)$ and $C_{Hi,t}(j)$ represent the domestic demand of cohort $j$ for a specific domestic product or a specific product from country $i$, respectively. $C_t(j)$ is a composite index, summarizing the

State-contingent claims, $T_t$ are real government taxes, and $\beta$ is the discount factor of the representative agent.

### Domestic human wealth
Domestic human wealth is defined as discounted stream of expected non-trade income conditional on survival:

$$
H_t(j) = \frac{1}{1-N_t(j)} \left[ \sum_{k=0}^{\infty} \frac{W_t}{P_t} N_t(j) - T_t(j) \right].
$$

In period $t$, the representative household chooses state-contingent sequences for consumption, leisure and share holdings to maximize intertemporal utility subject to budget constraints. Solving the Lagrangian associated with the optimization problem, the necessary first-order conditions may be stated as Eq. (2) and

$$
C_t(j) = \frac{W_t}{P_t} E_t \left[ 1 - N_t(j) \right],
$$

$$
F_{t+1} = \beta E_t \left[ \left( \frac{P_{t+1}}{P_t} \right)^{C_{t+1}(j)} + C_{t+1}(j) \right],
$$

where Eq. (4) represents the optimal intratemporal consumption-leisure decision and Eq. (5) is the Euler equation, describing the optimal intratemporal consumption pattern (the equilibrium stochastic discount factor for one-period ahead nominal payoffs equals the time-discounted stochastic growth in the marginal utility of consumption). Furthermore, Eq. (6) is the optimal intertemporal decision concerning the holdings of equity shares and equates the nominal price of a portfolio to its nominal expected payoff one-period ahead, discounted by $F_{t+1}$. This defines the stock-price dynamics. The first-order conditions ensure that at utility maximum the representative household cannot benefit from feasible intertemporal consumption reallocations.

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**Table 3**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
</tr>
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<tbody>
<tr>
<td>$\omega$</td>
<td>Elasticity of substitution between domestic and foreign goods</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Elasticity of substitutability between goods produced in foreign countries</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Elasticity of substitution between differentiated goods within one country</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Probability of dying</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Degree of openness</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Price rigidity</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Fraction of backward-looking price-setters</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Inverse steady-state Frisch elasticity of labor supply</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Conditional covariance between $F_{t+1}$ and $D_t^j$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Steady-state discount factor</td>
</tr>
<tr>
<td>$r$</td>
<td>Steady-state interest rate</td>
</tr>
</tbody>
</table>

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13 We abstract from money and model a cashless economy. This implicitly assumes the money supply adjusts to clear the market. Thus, we do not need to model money explicitly as here it simply functions as a unit of account.

14 We exclude the possibility of cohort members buying foreign shares. Empirically, this is an appropriate simplification since the Hong Kong and the US stock markets are highly correlated: Thus, introducing a foreign stock market into this study would likely add little insight.

15 Note, that new born individuals close their own cohort-specific insurance contract, redistributing the financial wealth among the survivors of their cohort. See Blanchard (1985) for details.

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demand for products from country \( i \), and \( C_{i,t}(j) \) is a composite index, summarizing the demand for products from all foreign countries.

Similar to the consumption indices, household’s minimization of total expenditures leads to the following producer price indices (PIPs) for domestic and foreign goods:

\[
P_{H,t} = \left[ \int_0^1 P_{i,t}(k)^{1-\varepsilon}dk \right]^{1-\varepsilon},
\]

\[
P_{F,t} = \left[ \int_0^1 P_{i,t}(k)^{1-\varepsilon}dk \right]^{1-\varepsilon},
\]

leading to the domestic CPI:

\[
P_t \equiv [1-\alpha]P_{H,t}^{1-\alpha} + \alpha P_{F,t}^{1-\alpha}.
\]

Following the preceding equations optimal allocation of any given expenditure yields the following demand equations

\[
C_{H,t}(j) = (1-\alpha)\left(\frac{P_{H,t}}{P_H}\right)^{-\alpha} C_i(j),
\]

\[
C_{F,t}(j) = \alpha \left(\frac{P_{F,t}}{P_F}\right)^{-\alpha} C_i(j),
\]

\[
t_t = \frac{P_{F,t}}{P_{H,t}} \left[ \int_0^1 P_{i,t}(k)^{1-\varepsilon}dk \right]^{1-\varepsilon},
\]

which can be approximated (up to a first-order) around a symmetric steady state satisfying \( t_i = 1 \) for all \( i \in [0,1] \) by \( t_i \equiv \log(\xi) = \int_0^1 s_idi \). Log-linearizing the domestic CPI under the assumption of a symmetric steady state satisfying the PPP yields

\[
p_t = p_{H,t} + \alpha \Delta s_t.
\]

Using the definition of the domestic PPI, there is a link to the domestic CPI:

\[
t_t = \frac{p_{F,t}}{p_{H,t}} + \alpha \Delta s_t.
\]

We see that the gap between producer and consumer price inflation is proportional to the change in the terms of trade, depending on the openness of the country.\(^{17}\)

Assuming that the LOOP holds on a brand level, we obtain \( P_{i,t}(k) = \mathbb{E}[P_{i,t}(k) | \forall i, k \in [0,1]) \), where \( P_{i,t}(k) \) represents the price of good \( k \) from country \( i \) measured in terms of country \( i \)'s currency. Integration over all products \( k \) yields \( P_{i,t} = \mathbb{E}[P_{i,t} | \forall i, k \in [0,1]) \), where \( P_{i,t} \equiv \left[ \int_0^1 P_{i,t}(k)^{1-\varepsilon}dk \right]^{1-\varepsilon} \) represents the composite price index of goods from country \( i \) measured in country \( i \)'s currency. A log-linearization of \( P_{i,t} \) around a symmetric steady state gives

\[
p_{i,t} = \int_0^1 (e_i + p_{i,t})di = e_t + p_{i,t},
\]

where \( p_t \) represents the log world price index and \( e \equiv \log(\Xi) \).\(^{18}\) Using this with the definition of the terms of trade gives \( s_t = e_t + p_t - p_{H,t} \).

Since the foreign PPI measured in foreign currency units is given by

\[
P_{F,t} = \left[ \int_0^1 (p_{i,t})^{1-\varepsilon}dk \right]^{1-\varepsilon},
\]

the assumption of the LOOP on a brand level, combined with identical preferences and the assumption of no home bias yields the PPP:

\[
P_{F,t} = \Xi p_{F,t},
\]

where \( P_{H,t} \) is defined in the same way as \( p_{F,t} \) and represents the domestic PPI measured in foreign currency units.

3.1.3. International risk sharing and the UIP

Assuming complete securities markets gross returns across countries should be equal and first-order conditions similar to those of the domestic country should hold in any country, leading to

\[
C_t = \xi C^*_{t+1}, \forall t
\]

where \( C_{t+1} \equiv \frac{P_{F,t+1}}{P_{H,t+1}} \) represents effective bilateral real exchange rate and \( \xi \) is a constant depending on initial conditions. Assuming (without loss of generality) symmetric initial conditions (implying zero net foreign assets) \( \xi = \lambda = 1 \). Taking logs and using the relationship between the terms of trade and the real exchange rate, we obtain

\[
c_t = c^*_t + (1-\alpha)s_t.
\]

To exclude arbitrage the nominal gross return \((1+r_t)\) on a safe one-period bond paying off one unit of currency in period \( t+1 \) must be defined as

\[
(1+r_t)\mathbb{E}[F_{t+1}^*] = 1.
\]

Assuming complete financial markets an analogous condition must be fulfilled in any foreign country:

\[
(1+r^*_t)\mathbb{E}[F_{t+1}] = 1.
\]

Defining the nominal effective exchange rate as \( \Xi_t = \int_0^1 \Xi_idi \), and combining Eqs. (13) and (14), we get the UIP

\[
\mathbb{E}[F_{t+1}]\left[ (1+r_t)-1-1+r^*_t\right] = 0,
\]

where \( r^*_t \) represents the interest rate for the rest of the world, i.e. \( (1+r^*_t)\equiv \int_0^1 (1+r^*_id) \). Log-linearizing Eq. (15) yields

\[
r_t-r^*_t = E_t(\Delta e_t + 1),
\]

which combined with the definition of the terms of trade directly leads to

\[
s_t = E_t s_{t+1} + (r^*_t-r_t)s_{t+1} - (r_t-r^*_t)\Xi_t s_{t+1}.
\]

\(^{16}\) To clarify the notation: \( P_{i,t} \) is the price index for domestic products, \( P_{r,t} \) is the price index for imported goods, and \( P_{i,t} \) is the price index for imported goods from country \( i \).

\(^{17}\) For \( \alpha = 0 \) we obtain the closed economy version and consumer and producer prices coincide.

\(^{18}\) Note that world CPI and PPI are the same as we assume that each country is of measure zero.
Gali and Monacelli (2005) point out that the UIP follows from international risk sharing and can be derived by combining the risk sharing condition with domestic and world Euler equations. Hence, it represents no independent equilibrium condition.19

Furthermore, Eqs. (13) and (5) lead to the conventional relationship between consumption growth and the interest rate.

\[
\hat{\beta} = \frac{1}{1 + r_t} = \beta E \left( \frac{P_t}{P_{t+1}} \right) \left( \frac{C_t}{C_{t+1}} \right)
\]

To rule out the possibility of Ponzi schemes we assume that the present value of financial wealth conditional on survival converges to zero:

\[
\lim_{k \to \infty} E_t \left\{ F_{t+k} + (1-\gamma)^k \Omega_t + \kappa(j) \right\} = 0.
\]

Using Eqs. (1) and (6) the budget constraint can be modified to a stochastic difference equation in financial wealth

\[
P_t C_t(j) + \varepsilon_t F_{t+j} + (1-\gamma) \Omega_{t+j} + \kappa(j) = W_t N_t(j) - P_t T_t + \Omega_t(j).
\]

Note that household optimization also implies that the equilibrium discount factor is given by

\[
F_{t+j} + \kappa = \hat{\beta}^j E \left( \frac{P_{t+j}}{P_t} \right) \left( \frac{C_t}{C_{t+j}} \right) = \prod_{i=0}^{k-1} F_t + i + 1.
\]

Using this with the no Ponzi condition, the definition of human wealth and the difference equation in financial wealth leads to

\[
P_t C_t(j) = [1 - \beta(1 - \gamma)] (\Omega_t(j) + H_t(j))
\]

where \(1 - \beta(1 - \gamma)\) represents the reciprocal of the propensity to consume out of financial and human wealth.20

3.1.4. Aggregation across cohorts

In order to facilitate aggregation across cohorts, we assume that the aggregate per-capita level of a specific variable \(G_t\) is given by the average of all generations, weighted with the cohort sizes: \(G_t = \sum_{j=-\infty}^{\infty} \gamma^j (1-\gamma)^j \Omega_t + \kappa(j)\). We first determine aggregate consumption and production.

3.1.4.1. Aggregate consumption and production. The intratemporal conditions yield

\[
C_{H,t} = (1-\alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\alpha} C_t, C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\alpha} C_t, C_{t+1} = \left( \frac{P_{t+1}}{P_t} \right)^{-\epsilon} C_{F,t}.
\]

where \(C_t\) can be derived from Eq. (7):

\[
C_t = \left( \frac{1}{1 - \alpha} \right)^{\frac{1}{\alpha-1}} (C_{H,t})^{\frac{1}{\alpha-1}} + \alpha \left( \frac{1}{\alpha} \right)^{\frac{1}{\alpha-1}} (C_{F,t})^{\frac{1}{\alpha-1}}
\]

Note here that the assumption of international risk sharing implies that \(C_t = C^{t}_t\) (∀t) equals world per-capita consumption.21

3.1.4.2. Aggregate FOCs. Since all equilibrium conditions are linear in cohort-specific variables, the aggregate conditions are simply given by

\[
\frac{C_t}{(1-N_t)} = \frac{W_t}{P_t},
\]

\[
P_t C_t + \varepsilon_t (F_{t+j} + \Omega_{t+j} + 1) = W_t N_t - P_t T_t + \Omega_t.
\]

\[
Q_t(k) = E_t \left( F_{t+j} + (1-\gamma)^j \Omega_t + D_t(k) \right)
\]

with aggregate nominal financial wealth given by

\[
\Omega_t = \beta_t\pi_t + \int_0^1 \Delta^t d\pi_t + \int_0^1 \pi_t (D_t(k) - \Delta_t) Z_t(k) \pi_t dk.
\]

We cancel out the probability of dying with the logical argument that the gross return on the insurance contract must equal one, since it has only redistributive effects.

From the first-order conditions we can derive a decision rule for optimal consumption. Combining Eqs. (23) and (24) describes the dynamic patterns of aggregate consumption, where the effect of financial wealth fades out as the probability of exiting the market goes to zero:

\[
\beta P_t C_t = \frac{\gamma}{(1-\gamma)} E_t \left( F_{t+j} + \Omega_{t+j} + 1 \right) + [1 - \beta(1-\gamma)] E_t \left( F_{t+j} + P_t + \gamma_{t+j} \right)
\]

3.2. Firms

3.2.1. Technology

For analytical tractability, we assume that a typical domestic firm produces a differentiated good \(k\) with a linear production function

\[
Y_t(k) = A_t N_t(k),
\]

where \(A_t \equiv \exp(a_t)\) represents a labor-augmenting productivity shock.22 Thus, cost minimizing leads to the following expression for firm’s real marginal costs:

\[
MC_t = (1-\bar{o}) \frac{W_t}{P_{H,t} A_t} \exp(\mu^p_t),
\]

where \(\bar{o}\) represents a subsidy optimally chosen by the government to correct the monopolistic distortion and \(\mu^p_t\) is a cost-push shock on marginal costs. Cost-push shocks can be justified, for example, by time-varying gaps between the natural level of output and the efficient one, which can be due to exogenous variations of price and wage markups, or fluctuations in labor tax income.

Using brand-specific demand functions and aggregating across domestic brands leads to

\[
Y_t \varphi_t = A_t N_t,
\]

where \(\varphi_t \equiv \int_0^1 \left( \frac{P_{H,t}(k)}{P_{H,t}} \right)^{-\epsilon} dk\) measures the relative price dispersion among domestic firms and \(N_t = \int_0^1 N_t(k) dk\) represents the aggregate per-capita amount of hours worked, which equals labor supply.

3.2.2. Price setting

Monopolistic firms are assumed to set prices in a Calvo-staggered manner. Calvo-pricing is a parable to account for persistence in...

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19 The appendix of Gali and Monacelli (2005) demonstrates that Eq. (17) can be solved forward, and the terms of trade are a function of current and anticipated real interest rate differentials: \(E_t \sum_{k=0}^{\infty} (\gamma^k - \epsilon E_t \nu_{t+k+1} - \kappa) \equiv (\epsilon + \kappa + E_t \nu_{t+k+1} - \kappa)\).


21 See the appendix of Di Giorgio and Nisticó (2007) for details.
inflation. A randomly selected fraction of firms \((1 - \theta)\) adjusts prices while the remaining fraction of firms \(\theta\) does not adjust. In addition, a fraction of \((1 - \tau)\) firms behaves in a forward-looking way and the remaining fraction \(\tau\) uses the recent history of the aggregate price index when they set prices. Thus \(\tau\) is a measure of the degree of backward-looking price-setting.

For the forward-looking fraction the optimization problem of a firm \(k\) has the same form as in Calvo (1983),

\[
\max_{\{p_{b,t}^k(k)\}} E_t \sum_{i=0}^{\infty} \beta^i \Delta_{t+i} \left[ \frac{p_{b,t}^k(k)}{p_{b,t+i}^k(k)} Y_t + i(k) - MC_t + Y_t + i(k) \right].
\]

(29)

and yields

\[
\left( \frac{p_{b,t}^k}{p_{b,t}^k} \right) = (1 + \mu) \frac{E_t \sum_{i=0}^{\infty} \beta^i \Delta_{t+i} \left[ \frac{p_{b,t}^k(k)}{p_{b,t+i}^k(k)} Y_t + i(k) - MC_t + Y_t + i(k) \right]}{E_t \sum_{i=0}^{\infty} \beta^i \Delta_{t+i} \left[ \frac{p_{b,t}^k(k)}{p_{b,t+i}^k(k)} Y_t + i(k) - MC_t + Y_t + i(k) \right]},
\]

(30)

where \((1 + \mu) = \left( \frac{e^{\alpha \tau}}{1 - e^{\alpha \tau}} \right)\) represents the steady-state markup over a weighted average of expected marginal costs and \(\Delta_{t+i}\) represents the real discount factor.

Defining the domestic index for the prices newly set in period \(t\) \((\tilde{p}_{b,t}^k)\) as a weighted average of the forward- and backward-looking prices, and assuming a rule of thumb for the backward-looking price-setters,

\[
\tilde{p}_{b,t}^k = \pi_{b,t-1} + \pi_{t-1},
\]

(31)

the following equations can be derived using some straightforward algebra:

\[
\tilde{p}_{b,t} = \left[ \alpha \phi_{b,t-1}^{\pi_{b,t-1}} + (1 - \alpha) \left( \frac{p_{b,t-1}^{\pi_{b,t-1}}} {p_{b,t-1}} \right) \right]^{1 / \alpha}.
\]

(32)

\[
\tilde{p}_{b,t} = (1 - \tau) \phi_{b,t} + \tau \phi_{b,t},
\]

(33)

where \(\pi_{b,t} = \ln \left( \frac{\tilde{p}_{b,t}}{\tilde{p}_{b,t-1}} \right)\) represents the domestic producer price inflation.

3.3. Equilibrium

One virtue of the model presented here is its tractability, which is beneficial in terms of both the type of analysis that can be undertaken and the intuitive nature of the results. As the model can be solved for an infinite number of steady states, we place a restriction on the initial equilibrium. Without loss of generality, we assume a baseline symmetric steady state with equal and constant consumption and a real exchange rate of value one. Together with PPP, this also implies a value of one for the terms of trade. In what follows, lower-case letters are used for describing percentage deviations from equilibrium values of their upper-case counterparts.

3.3.1. The demand side

A clearing of the goods market for each good \(k\) of the domestic small open economy requires

\[
Y_i = C_{k,t}(k) + \int_0^{\gamma} C_{k,t}(k) dl = \left( \frac{p_{k,t}(k)}{p_{k,t}} \right)^{\gamma} \left( 1 - \alpha \right) \left( \frac{p_{k,t}(k)}{p_{k,t}} \right)^{-\gamma} C_t + \alpha \int_0^{\gamma} \left( \frac{p_{k,t}(k)}{p_{k,t}} \right)^{-\gamma} \left( \frac{p_{k,t}(k)}{p_{k,t}} \right)^{-\gamma} C_t dl.
\]

(34)

for all \(k\in[0,1]\) and all \(t\), where \(\partial_{k,t}(k)\) represents country \(k\)’s demand for the domestic good \(k\). Taking this together with the definition of aggregate output gives

\[
Y_i = C_{k,t}^\alpha \left[ \left( 1 - \alpha \right) \alpha \int_0^{\gamma} \left( \frac{S_i}{S_t} \right)^{\gamma-\gamma} dl \right],
\]

where \(S_i\) represents the effective terms of trade of country \(i\) \((P_{k,t}/P_{k,h,t})\). Following \(\int_0^{\gamma} S_t dl = 0\), we get an approximation around a symmetric steady state

\[
y_t = c_t + \alpha \omega_t,
\]

(35)

where \(\omega = \zeta + (1 - \omega)(\alpha - 1)\). Since an analogous condition holds for each country in the world, it follows that \(y_t = c_t\). Furthermore, international risk sharing implies

\[
y_t = y_t^* + \frac{1}{c_{\alpha}} s_t,
\]

(36)

where \(c_{\alpha} = \frac{1}{(1 - \alpha) + \alpha \sigma}\).

The indices of average real dividend payments and the average stock market capitalization are

\[
D_t = \frac{1}{P_t} \int_0^{\gamma} D_i(k) dl, Q_t = \frac{1}{P_t} \int_0^{\gamma} Q_i(k) dl. \]

Under the assumption of a balanced world government budget, the equilibrium aggregate world net supply of state-contingent bonds must equal zero. However, in an open-economy model, we can approximate around any given distribution of bonds across countries, and thus assume without loss of generality that bond holdings of domestic and foreign households are zero in the initial equilibrium:

\[
B_t + \int_0^{\gamma} \int_0^{\gamma} E_t F_i dl = \frac{1}{P_t} B_i(t) + \int_0^{\gamma} B_i(t) dl = 0.
\]

Using this condition along with Eq. (25), the equilibrium present discounted nominal value of future aggregate nominal financial wealth is equal to the current level of domestic nominal stock prices:

\[
E_t \left\{ F_{1,t} + 1 \Omega_t + 1 \right\} = \int_0^{\gamma} Q_t(k) dl = Q_t,
\]

where we have used the intertemporal condition (Eq. (6)) and the logical argument that \(\int_0^{\gamma} Q_t(k) dl = 1.25\).

Moreover, the demand side is constrained by

\[
D_t = \frac{P_{k,t} Y_t}{1 - \beta} \frac{Y_t}{Y_t} W_t N_t. \]

(37)

Let \(\Pi_t\) denote the domestic gross inflation rate. Integrating over all firms, we thus get the demand side of the domestic economy, characterized by the following Euler equation:

\[
\beta C_t = \frac{\gamma}{(1 - \gamma)} Q_t + \left[ 1 - \beta(1 - \gamma) \right] E_t \left\{ F_{1,t} + 1 \Omega_t + 1 \right\}, \]

and

\[
Q_t = E_t \left\{ F_{1,t} + 1 \Omega_t + 1 \left[ Q_t + 1 + D_t + 1 \right] \right\}. \]

23 For the derivation of Eq. (34), we use Eq. (21) and the assumption of symmetric preferences across countries, leading to \(\partial_{k,t}(k) = \alpha \left( \frac{p_{k,t}}{p_{k,t}} \right)^{-\gamma} \left( \frac{p_{k,t}}{p_{k,t}} \right)^{-\gamma} C_t. \)

24 See Gall and Monacelli (2005) for a detailed derivation.

25 The domestic country is a small country of measure zero, so the bilateral exchange rates in the rest of the world necessarily sum up to one.
Furthermore, using $E(xy) = E(x)E(y) + \text{Cov}(x,y)$ we derive

$$Q_t = E\left\{\left[F_{t,t+1} + Q_{t,t+1} + \frac{D_t}{Q_t}\right] - Q_t\right\},$$

where $\varepsilon$ represents the conditional covariance between the stochastic discount factor and the nominal gross return rate on stocks. This defines the following risk premium:26

$$EP_t = E\left\{\left[\frac{Q_{t,t+1} + \frac{D_t}{Q_t}}{Q_t}\right] - (1 + r_t)\right\} = (1 + r_t)\varepsilon.$$

Log-linearizing the demand side of the model gives

$$y_t = \frac{1}{1 + \Psi}E_t y_{t+1} + \frac{\Psi}{1 + \Psi}q_t - \frac{1}{1 + \Psi}(r_t - E_t \pi_t + 1 - \rho),$$

$$= \frac{1}{1 + \Psi}E_t \{\Delta s_{t+1} + 1\},$$

$$w_t - \eta_t = c_t + \varphi n_t,$$

$$q_t = \frac{\beta}{1 + \varepsilon}E_t q_{t+1} + \frac{1 + \varepsilon - \beta}{1 + \varepsilon}E_t d_t + 1 - (r_t - E_t \pi_t + 1 - \rho),$$

$$d_t = \frac{Y}{D} (p_t - p_{t+1} + \varepsilon)^{-\frac{\alpha}{\Phi}} (n_t + w_t - p_t),$$

where $\Psi = \frac{1}{1 - \gamma}$ is the inverse of the steady-state Frisch elasticity of labor supply, $\Psi = \frac{\gamma(1 - \gamma)}{1 - \gamma} / (\Omega - \sigma)$ and $\rho = -\log(\beta)$.

3.3.2. The supply side

Equilibrium in the labor market and the definition of the terms of trade imply

$$MC_t = \frac{C_t P_t \exp(\mu_t^p)}{A_t - y_t P_t \Gamma_{HH}}.$$

Log-linearizing this by using the assumption of a linear production technology, the optimal labor-leisure decision of the households and the risk sharing condition, equivalence real marginal costs are given by

$$\log(MC_t) = y_t^* + \varphi y_t - (1 + \varphi) a_t + s_t + \mu_t^p$$

$$= (\alpha_\sigma + \varphi) y_t + (1 - \alpha_\sigma) y_t - (1 + \varphi) a_t + \mu_t^p.$$  

In a flexible-price equilibrium all firms set their prices equal to a constant markup over marginal costs $\mu$. Thus marginal costs reach their long-run level at each point in time ($mc_t = 0$) and the flexible-price equilibrium level of output follows directly from the preceding equation:

$$y_t^* = \Gamma_p a_t - \alpha_\Gamma y_t^*,$$

where $\Gamma_p = (1 + \varphi) / (\alpha_\sigma + \varphi)$ and $\Gamma = 1 - (\varphi - 1) / (\alpha_\sigma - 1)$. This links real marginal costs with the output gap $x_t^* = (y_t^* - y_t^*)$:

$$mc_t = (\alpha_\sigma + \varphi) x_t^* + \mu_t^p.$$  

Log-linearizing the price-setting equations (Eqs. (31)–(33)) gives the conventional hybrid New Keynesian Phillips curve:

$$\pi_{HH} = \Phi(\theta \pi_{HH} + 1 + \tau_{HH-1}) + \lambda mc_t^* + \epsilon_t^s,$$

where $\lambda = (1 - \tau)(1 - \tilde{\beta}) - 1 + \tilde{\beta}0$, $\Phi = (\theta + \tau(1 - \tilde{\beta}))^{-1}$ and $\epsilon_t^s = \lambda H t^2$.

3.4. The natural rate and the canonical representation

Exchange-rate stability is enshrined as the objective of monetary policy in Hong Kong. We therefore close the model by assuming an exchange-rate peg: $\varepsilon_t = 0$. Hence, monetary policy is implicitly given, since the interest rate adjusts endogenously to ensure a fixed exchange rate. Under a flexible regime, exchange-rate dynamics were given by $\Delta s_t = \Delta s_{t+1} = \pi_{HH}$. Following an exchange-rate peg, the central bank has to manage demand in a way compatible with $\Delta s_t = -\pi_{HH}$ (recall that the terms of trade are proportional to the gap between domestic and foreign supplies due to Eq. (36)).

As usual in New Keynesian models, we can summarize the model by a few equations, representing percentage deviations of the state variables from their steady-state values. Furthermore, it is common in the literature to provide a structural interpretation to shocks.

The reduced system is given by

$$x_t = \frac{\sigma_a}{\gamma_0} E_t x_{t+1} + \frac{\Psi}{\gamma_0} \tilde{a}_t - \frac{1}{\gamma_0} (r_t - E_t \pi_{HH} + 1 - \tau_{HH}^s),$$

$$\hat{q}_t = \frac{\beta}{1 + \varepsilon} E_t \hat{q}_{t+1} - \frac{\lambda}{1 + \varepsilon} E_t x_{t+1} + 1 - (r_t - E_t \pi_{HH} + 1 - \tau_{HH}^s) + \eta_b,$$

$$\pi_{HH} = \delta(\theta \pi_{HH} + 1 + \tau_{HH-1} + \kappa_a x_t + \epsilon_t^s),$$

$$\eta_t = \rho_\eta \eta_{t-1} + \epsilon_t^\eta,$$

where $\delta = \Psi / (1 - \gamma) / (\Omega - \sigma)$, $\lambda = 1 - (1 - \tilde{\beta}) (1 + \varphi - \mu_t^p)$ and $\Omega = \lambda (\sigma_\sigma + \varphi) / (1 - \gamma) / (\Omega - \sigma)$. $\eta_b$ is a shock accounting for all non-fundamental movements in the stock-price gap.27 Furthermore, the natural rate, summing up all exogenous variables in the dynamic IS equation, is given by

$$\tau_{HH} = \rho_\pi \tau_{HH-1} + \epsilon_t^\pi,$$

$$y_t^* = \rho_\pi y_t^* + \epsilon_t^\pi,$$

$$\eta_b = \rho_\eta \eta_{b-1} + \epsilon_t^\pi,$$

and the $\epsilon$'s are normally distributed with constant mean and variance $\sigma^2$. Finally, following Nisticò (2005), the natural stock-price index is given by $\tilde{q}^2 = \tilde{y}^2$.

The reduced-form of the DSGE model is founded on structural parameters that describe the optimal behavior of firms and households. These parameters can be considered independent of the policy regime, and are thereby not subject to the Lucas critique.

4. Estimation and model fit

Having a new general equilibrium model for Hong Kong is important in its own right, but it is also important to investigate the mapping of the model into reality using all available data. We use a combination of less formal calibration and estimation in parameterizing the model.

27 See the Appendix A for details.
The parameters of the DSGE model are estimated by Bayesian methods, conditional on prior information concerning the values of parameters. Following standard practice, we have removed trend components from the observed macroeconomic variables using the HP-filter.\textsuperscript{38} Striking advances have been made in recent years in estimating DSGE models, shifting emphasis in quantitative macroeconomics from calibration exercises to directly estimating the parameters of a structural model and letting the data speak. The Bayesian technique—as forcefully claimed by An and Schorfheide (2007) and Fernández-Villaverde (2009)—is now the standard tool for estimation of DSGE models. Linear approximation methods lead to a state-space representation of the DSGE model that can be analyzed with a Kalman filter. Together with the specification of prior distributions for the parameters, the state-space representation can be translated to form the posterior distribution. Bayesian estimation looks for the parameter that maximizes the posterior, given the prior and the likelihood based on the data. In order to obtain numerically a sequence from the unknown posterior distribution, we employ the Metropolis–Hastings Markov–Chain Monte Carlo algorithm with 250,000 draws (neglecting the first 10,000). From the computational point of view, the Dynare toolbox for Matlab has been applied.

For the estimation of models (10), (36), (45), and (48)–(54), $x_t = y_t - y_d^* + \Delta c_t - \Delta d_{t,t-1}$ are used, where the last equation follows from the assumption of a small economy with a fixed exchange rate. We employ quarterly data for 4 observables for the sample 1981Q1–2007Q3: the real GDP of Hong Kong, the Hang Seng index, the consumer price index of Hong Kong and US GDP.\textsuperscript{29} The last series is used as a proxy for foreign demand ($y_t^*$).

4.1. Calibration

A number of parameters remain fixed throughout the estimation procedure. This is because they are either notoriously difficult to estimate (in the case of substitution elasticities between home and foreign goods and the Frisch wage elasticity of labor supply) or they are better identified using other information. To account for their influence on the estimation results we provide an extensive robustness analysis, using different values for these parameters.\textsuperscript{30} In what follows, we present the baseline calibration of the fixed parameters.

The steady-state interest rate is set to 0.01, implying a long-run nominal interest rate of 4% annually, and the discount factor $\beta$ is set to 0.995. Following Nisticò (2005), the markup over marginal costs is $0.13$, implying an annualized steady-state risk premium of 0.015, which is a widely used value in the New Keynesian Phillips Curve and found a rather high value for the degree of backward-looking price-setting of 0.5.\textsuperscript{34} Given the volatility of inflation in Hong Kong, we use a smaller prior for the inflation persistence parameter $\tau$ of 0.2, but also report estimates for a higher value. The degree of openness is bounded between 0 (autarky) and 1 (complete integration). We use a value of 0.5, which seems at least plausible for a small open economy like Hong Kong. The persistence parameters for the AR(1) processes of $a_t$, $e_t$ and $\eta_t$ are all set to the standard value of 0.7 and the standard deviation of all shocks is set to 2. These assumptions ensure that the model is roughly consistent with the autocovariance patterns observed in the data. Since all parameters besides the standard deviation of shocks are bounded by the unit interval $[0, 1]$, we use Beta distributions. For the standard deviations we use inverse Gamma distributions as is the standard convention.

<table>
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<tr>
<th>Prior distribution Type</th>
<th>Mean</th>
<th>St. dev.</th>
<th>Posterior distribution Mean</th>
<th>St. dev.</th>
<th>Conf. int.</th>
<th>Stand. coeff.</th>
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</tr>
<tr>
<td>$\rho_y$ Beta 0.7 0.1 0.8306 0.0372 [0.7690,0.8909] 22.33</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{\eta}$ Beta 0.7 0.1 0.7759 0.0522 [0.6940,0.8623] 14.84</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_a$ Gamma 2.0 2.0 2.6684 0.1847 [2.3632,2.9821] 14.45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_y$ Gamma 2.0 2.0 0.6148 0.0412 [0.5446,0.6831] 14.92</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_{\eta}$ Gamma 2.0 2.0 0.9667 0.1253 [0.7588,1.1660] 7.72</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_{\eta}$ Gamma 2.0 2.0 2.9396 0.6084 [1.9298,3.9490] 4.83</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.2. DSGE prior and posterior distributions

Detailed descriptions of the prior distributions for the structural DSGE parameters are summarized in Table 4. In selecting the prior distributions for the parameters to be estimated, we are guided by conventions in the literature. The key parameter $\gamma$, Nisticò (2005) uses a rather small value of 0.03, which corresponds to a financial planning horizon of slightly more than 8 years in a quarterly model, and is in line with estimates of Milani (2008), who derives a planning horizon of 10 years. However, in a more recent study Castelnuovo and Nisticò (2010) found a value of 0.13 for the US. We found this value to be more convincing as it refers to a stronger consumption-wealth effect and a more realistic financial planning horizon of slightly less than 2 years. For a sensitivity analysis of the results we also use their posterior bands $[0.07,0.19]$. Price rigidity is set to the standard value of 0.7, which is close to the estimates of Christiano et al. (2005) for the US or Genberg and Pauwels (2005) for Hong Kong. The latter study estimates a single open-economy New Keynesian Phillips Curve and found a rather high value for the degree of backward-looking price-setting of 0.5.\textsuperscript{34} Given the volatility of inflation in Hong Kong, we use a smaller prior for the inflation persistence parameter $\tau$ of 0.2, but also report estimates for a higher value. The degree of openness is bounded between 0 (autarky) and 1 (complete integration). We use a value of 0.5, which seems at least plausible for a small open economy like Hong Kong. The persistence parameters for the AR(1) processes of $a_t$, $e_t$ and $\eta_t$ are all set to the standard value of 0.7 and the standard deviation of all shocks is set to 2. These assumptions ensure that the model is roughly consistent with the autocovariance patterns observed in the data. Since all parameters besides the standard deviation of shocks are bounded by the unit interval $[0, 1]$, we use Beta distributions. For the standard deviations we use inverse Gamma distributions as is the standard convention.

\textsuperscript{38} As an alternative, a DSGE model incorporating deterministic or stochastic trends can be estimated. However, as discussed in Clements and Hendry (1999), intermittent structural breaks render such trends empirically inadequate representations of low-frequency variation in observed macroeconomic variables.

\textsuperscript{39} It is worth noting that Genberg (2005) has found in a VAR framework that Hong Kong’s output growth depends to a large extent on developments in the US.

\textsuperscript{30} An additional rationale for an extensive robustness analysis is that for most of these parameter values there are no estimates available for Hong Kong. So we have to take estimates from the US, which might be not appropriate.

\textsuperscript{31} Obstfeld and Rogoff (2000) have shown that such high elasticities can explain an observed large home bias in trade.

\textsuperscript{32} For instance, King and Rebelo (1999) use a value of 4.

\textsuperscript{33} We need a value for the steady-state consumption to pin down $\frac{c}{y}$ which determines $\gamma$. It is easily shown that $\frac{c}{y} = \frac{1}{1 + \frac{1}{\gamma} - \frac{1}{\tau}}$. See Nisticò (2005) for details.

\textsuperscript{34} A similar value has been found by Funke (2006) for mainland China.

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The estimated standard deviations of the shocks give a first indication on what shocks seem to drive the cyclical variations in our macroeconomic time series. The shocks estimated to have the highest standard deviations are productivity and stock-price gap shocks. A more detailed analysis of the driving forces of the Hong Kong business cycle will be given in the next section.

4.3. Properties of the estimated model

In this section, we turn to the results from the model, once parameters are obtained from maximizing the posterior over the entire sample period. We do so in three steps. First, in Fig. 2 we show the one-step ahead predictions of the model. Obviously, a prerequisite for an estimated DSGE model is a proper empirical representation of the data generating process. The overall impression is that the model is able to mimic the time series, i.e. the results indicate that the model can match the cyclical properties of the data very well, with minor difficulties in replicating the peaks in inflation and the output gap.

In a second step, Table 5 presents the asymptotic forecast error variance decomposition for the estimated posterior means. Overall, the results clearly provide evidence, that output, inflation and the natural interest rate are substantially driven by productivity disturbances, while non-fundamental movements in stock prices are the main driver of the stock-price gap and the nominal interest rate. Perhaps surprisingly for a small open-economy oscillations in the output gap are only driven to a very limited extent by foreign demand. Inspecting the results for the natural rate, which depends only on productivity and foreign demand by construction, we obtain a very similar result. Although we found a considerably high degree of openness, foreign demand shocks seem to be of secondary importance. And the same is true for cost-push shocks, which explain one third of the observed inflation volatility, but are of less importance for the volatility of any other variable.

solid line: posterior; dashed line: prior; vertical line: posterior mean

Fig. 1. Prior vs. posterior distributions in the Metropolis–Hastings procedure.
Turning to the stock-price gap shock, the decomposition supports the findings of Table 4. Stock-price gap shocks account for nearly the complete volatility of the stock-price gap and the nominal interest rate. Modeling an economy with a fixed exchange rate and a consumption-wealth effect, it is clear that the nominal rate has to be adjusted to eliminate pressures on the exchange rate. Any movement in the stock-price gap would have an effect on demand and hence implies a corresponding response on the nominal interest rate. As long as Hong Kong's currency board arrangement remains in place, Hong Kong will follow the broad direction of US interest rate moves. However, US interest rate movements are not the sole guiding force. Hong Kong's interest rate adjustment mechanism is an automatic system that maintains the stability of the HK dollar exchange rate. Under Hong Kong's currency board system, it is interest rates rather than the exchange rate which adjust to inflows or outflows of funds. The monetary base increases when the US dollar is sold to the currency board for the domestic currency (capital inflow). It contracts when the foreign currency is bought from the currency board (capital outflow). The expansion or contraction in the monetary base causes Hong Kong's interest rates to fall or rise respectively, creating the monetary conditions that automatically counteract the original capital movement, while the exchange rate remains stable.

Because Hong Kong allows the size of the monetary base to be determined by the inflow and outflow of capital, through a non-discretionary undertaking to buy and sell HK dollars for US dollars at a fixed exchange rate, Hong Kong is facing volatile interbank interest rates and, as a consequence, volatile deposit and lending rates for consumers. Next we turn to the historical contribution of each type of shock over the sample period. A historical shock decomposition for the output gap, inflation and the Hang Seng index is given in Fig. 3, whereas Fig. 4 shows the decomposition for the nominal and the natural interest rate. Analyzing these figures can be a very fruitful exercise, since it allows to identify the main sources of specific booms and recessions. Overall the results confirm our previous findings. Nevertheless, the graphs indicate that foreign demand shocks had a stronger influence on the output gap and the nominal interest rate during the 1980s and early 1990s. In addition, foreign demand has a substantial effect on the nominal interest rate over the whole sample period, although Table 5 suggests a more or less negligible asymptotic effect. On the inflation side, we observe that periods of extremely high or low inflation rates are mainly caused by cost-push shocks. Summing up, we found overwhelming evidence that productivity is the main driver of cyclical variations in output and prices, whereas the non-fundamental movements in share prices largely explain the variations in the Hang Seng index and the nominal interest rate. Moreover, foreign demand has an important influence on the central bank's policy.

The Bayesian estimation results indicate that agents exhibit a positive elasticity of consumption to share prices of $\Psi_T \approx 0.0647$. How big are these spillovers from the stock market? From an economic point of view, this value indicates a non-negligible positive wealth effect. Overall, we think that the preceding analysis supports this view. On a cautionary note, however, we have to acknowledge, that the estimations could depend strongly on the prior distribution and the calibrated parameters. Hence, the following section examines

![Fig. 2. In-sample one-step ahead predictions of the estimated model.](image)

<table>
<thead>
<tr>
<th>Productivity</th>
<th>Foreign demand</th>
<th>Cost-push</th>
<th>Stock-price gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>75.58</td>
<td>11.39</td>
<td>13.04</td>
</tr>
<tr>
<td>$\delta$</td>
<td>1.87</td>
<td>0.33</td>
<td>0.38</td>
</tr>
<tr>
<td>$r$</td>
<td>2.95</td>
<td>4.19</td>
<td>1.79</td>
</tr>
<tr>
<td>$\Pi^m$</td>
<td>96.33</td>
<td>3.67</td>
<td>0.00</td>
</tr>
<tr>
<td>$\pi$</td>
<td>57.16</td>
<td>7.97</td>
<td>34.87</td>
</tr>
<tr>
<td>$s$</td>
<td>84.32</td>
<td>10.02</td>
<td>5.66</td>
</tr>
<tr>
<td>$y$</td>
<td>90.50</td>
<td>3.43</td>
<td>6.07</td>
</tr>
<tr>
<td>$y^*$</td>
<td>99.76</td>
<td>0.24</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 5
Forecast error variance decomposition.

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the robustness of the wealth effect, using different calibrations and different assumptions on prior distributions.

4.4. Sensitivity analysis

In Table 6 we provide an extensive sensitivity analysis of our estimation results by comparing posterior means and confidence intervals for different assumptions on calibrated parameters and prior distributions. In addition, Bayesian estimations provide a straightforward way of comparing different models by their marginal likelihood, which is given in the last line of the table. The first two columns show the results for our baseline model and a reference model without any wealth effects ($\gamma = 0$). The subsequent 5 columns use different values for the inverse of the Frisch elasticity of labor supply $\phi$, and the substitution elasticities between domestic and foreign goods $\varpi$, and goods produced in different foreign countries $\zeta$, respectively. In the last two columns we present estimations using the posterior bounds of Castelnuovo and Nisticó (2010) as prior means for $\gamma$ and a mean of 0.5 for the degree of backward-looking price-setting, as found in Genberg and Pauwels (2005).

Comparing our perpetual-youth approach with a standard New Keynesian model confirms our findings, since the remaining estimated parameters do not change substantially. Concerning the different calibration of $\phi$, $\varpi$ and $\zeta$ there are three key findings: Firstly, estimations of the persistence parameter for all shocks and the standard deviation of the foreign demand shock are remarkably robust. Secondly, by increasing the inverse of the Frisch elasticity, estimations of the degree of openness increase, while for increasing values of substitution elasticities, the degree tends to fall. The reason for both effects is obviously the influence of foreign demand on the domestic economy. Higher values for the substitution elasticities increase the effect of foreign demand, and consequently estimations of the degree of openness for a given dataset have to fall. And the same is true for the Frisch elasticity, which captures the elasticity of hours worked to the real wage rate.
fall. Thirdly, for higher substitution elasticities the degree of price rigidity increases, whereas the standard deviation of productivity falls, due to the effect on the estimated degrees of openness and the subsequent influence on natural output and the New Keynesian Phillips curve.

Assuming different priors for $\gamma$ has no considerable effect on the remaining parameter estimations, except for the standard deviation of the stock-price gap shock which sharply increases for the scenario with strong wealth effects. However, the choice of the prior mean has a significant effect on the estimated $\gamma$, indicating that this value seems to be crucial for determining the wealth effect. Overall, marginal likelihoods suggest that our baseline estimation fits the data better. The higher prior mean for $\tau$ leads only to a slightly higher estimation of the degree of backward-looking price-setting without a significant change in any other parameter. The fall in the posterior mean and the lower marginal likelihood indicate that our baseline choice seems to be more appropriate. Comparing the marginal likelihoods of the different specifications shows nearly no difference between our model and the standard model, but also suggests that our model outperforms any other specification. Summing up we can conclude that our baseline estimations are quite robust, although we cannot infer a significant improvement in terms of the marginal likelihood with respect to the reference New Keynesian model.

Another widely applied method to assess the validity of the estimated DSGE model is to compare the standard deviations and correlation between the variables of the data and simulations of the different model specifications using the corresponding posterior means. The results of this exercise are given in Table 7. Obviously, a small-scale model as the one used in this paper cannot capture all correlations given in the data exactly. Comparing our approach to the reference model shows that our model performs better with respect to correlations between the stock-price gap and the output gap, and the stock-price gap and inflation, respectively. Concerning the different calibrations the table shows that high values for $\varphi$ and $\varpi/\zeta$ lead to an implausible negative correlation between the output gap and the stock-price gap. Moreover, a low prior mean for $\gamma$ overestimates this correlation in a comparable manner as the reference model. Proposing a high mean on the other hand leads again to an implausible negative value. In a nutshell, we can conclude that a more detailed modeling of stock markets in order to improve the implied second-order moments is a promising way to go, but the small-scale model presented here seems to be one step in the right direction.

Having shown that the model fits the data reasonably well, we next elucidate how the variables react in response to the structural shocks. Figs. 5–8 single out the impulse response functions. We present these graphs as they nicely illustrate the functioning of our modelling toolkit.

4.5. Impulse response functions

How do share prices interact with output, inflation, and monetary policy? For the US several papers analyze the question whether monetary policy has an effect on stock prices, and whether the Fed should use their power to manage financial shocks. For a fixed exchange-rate regime like Hong Kong the question should rather be: Do unexpected movements in stock markets affect the monetary policy, which is consistent with the peg? The preceding analysis already indicates that this seems to be the case. In the following we

---

38 Note that the estimations of the standard deviation of the cost-push shock move in line with estimations of $\alpha$, since both affect the New Keynesian Phillips curve.

39 See, for example, Lee (1992), Thorbecke (1997), Bernanke and Kuttner (2005) and Bjørnland and Leitemo (2009) for a discussion.

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### Table 6
Comparison of estimated parameters.

<table>
<thead>
<tr>
<th>Baseline</th>
<th>( \gamma = 0 )</th>
<th>Different calibrations</th>
<th>Different prior means</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma )</td>
<td>0.1300</td>
<td>0.1282</td>
<td>0.1303</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>[0.0003, 0.2713]</td>
<td>[0.0005, 0.2654]</td>
<td>[0.0000, 0.2673]</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.1234</td>
<td>0.1235</td>
<td>0.1154</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.6658</td>
<td>0.6619</td>
<td>0.6440</td>
</tr>
<tr>
<td>( \mu_{\theta} )</td>
<td>0.5333, 0.7988</td>
<td>0.5318, 0.7942</td>
<td>0.5052, 0.7849</td>
</tr>
<tr>
<td>( \rho_\theta )</td>
<td>0.8836</td>
<td>0.8818</td>
<td>0.8838</td>
</tr>
<tr>
<td>( \mu_{\rho_\theta} )</td>
<td>0.5448, 0.6831</td>
<td>0.7716, 0.8931</td>
<td>0.7702, 0.8927</td>
</tr>
<tr>
<td>( \rho_\eta )</td>
<td>0.7750</td>
<td>0.7734</td>
<td>0.7816</td>
</tr>
<tr>
<td>( \sigma_\theta )</td>
<td>0.6940, 0.8623</td>
<td>0.6888, 0.8569</td>
<td>0.7015, 0.8665</td>
</tr>
<tr>
<td>( \sigma_\eta )</td>
<td>0.2684 ( \times 10^{-4} )</td>
<td>2.8676</td>
<td>2.3878</td>
</tr>
<tr>
<td>( \gamma_\theta )</td>
<td>0.2363 ( \times 10^{-2} )</td>
<td>2.3594 ( \times 10^{-2} )</td>
<td>2.1490, 2.6662</td>
</tr>
<tr>
<td>( \gamma_\eta )</td>
<td>0.5448, 0.6831</td>
<td>0.5464, 0.6836</td>
<td>0.5450, 0.6850</td>
</tr>
<tr>
<td>( \sigma_\eta )</td>
<td>0.9667</td>
<td>0.9589</td>
<td>1.0924</td>
</tr>
<tr>
<td>( \tau_\theta )</td>
<td>0.7588, 1.1660</td>
<td>0.7550, 1.1567</td>
<td>0.8582, 1.3215</td>
</tr>
<tr>
<td>( \tau_\eta )</td>
<td>2.0396</td>
<td>2.8220</td>
<td>2.8910</td>
</tr>
<tr>
<td>( \beta )</td>
<td>1.9298, 3.9490</td>
<td>[1.8000, 3.8182]</td>
<td>[1.8892, 3.8828]</td>
</tr>
</tbody>
</table>

Marginal likelihood:

| Marginal likelihood | \(-799.46\) | \(-798.93\) | \(-801.18\) | \(-804.59\) | \(-808.18\) | \(-801.35\) | \(-813.57\) | \(-802.81\) | \(-812.30\) | \(-803.24\) |
will analyze this question more explicitly and show that our model also implies significant reactions of stock prices to productivity and foreign demand disturbances.

4.5.1. Productivity shock

In Fig. 5 the impulse responses after a productivity shock are shown (the dashed lines represent the 95% interval). Apart from the effect of stock-price dynamics on consumption and the restrictions on monetary policy due to the exchange-rate peg, the responses show the typical characteristics (see e.g. Gall, 2008). An increase in the productivity index leads to an increase in natural output and, as a result, a negative output gap. The productivity shock reduces real marginal costs, enabling firms to lower producer prices. The rise in output leads to an increase in consumption, whereas the negative output gap implies a fall in employment and nominal wages at impact. In a standard New Keynesian model, the central bank would move to accommodate the improvement in technology by strongly decreasing the interest rate. In the case of an exchange-rate peg, however, the central bank’s hands are tied and the interest rate reaction is tiny (but significant) and only in order to cope with the downward pressure on the exchange rate (appreciation). This pressure is due to the positive effect on the terms of trade, stemming from the diminishing producer prices. In the medium term, actual output catches up and the output gap closes. The increase in productivity implies an increase in the natural stock prices, while output catches up and the output gap closes. The increase in the natural level of output strengthens the increase in the output gap. Natural share prices follow the natural level of output, and the natural stock-price gap show the same dynamics. To eliminate the effect on consumption-wealth channel. The very strong and highly significant response of the monetary authority is compatible with the findings of Fig. 4, illustrating that foreign demand is one key driver of the nominal interest rate.

4.5.2. Foreign demand shock

To understand the reactions after a foreign demand shock, given in Fig. 6, it is crucial to clarify the effect of an increase in foreign demand on the natural level of output. Natural output is consistent with the flexible price allocation, implying a constant steady-state markup over marginal cost (the desired markup of firms with monopolistic power). As shown in Eq. (44) the influence of foreign demand on real marginal cost is twofold. The direct effect is due to the assumption of international risk sharing, leading to an increase in consumption. The so-called wealth effect increases real wages and hence marginal costs through the optimal labor-leisure decision of optimizing households. The indirect effect works through the terms of trade channel. The appreciation of the terms of trade implies a lower product real wage (\( \pi_t - \pi_{Pt,L} \)) and lowers real marginal cost. Given a log-utility function and low substitution elasticities \( \sigma \) and \( \xi \), the wealth effect dominates and foreign demand increases marginal cost. Consequently, the natural level of output has to fall to restore the flexible-price allocation of a constant steady-state markup.

A shock to foreign output increases domestic output and consumption due to the boost in demand. Since this is accompanied by a fall in marginal utility, firms seeking to boost production and hire more workers have to increase wages to achieve an increase in hours worked. As a result, real marginal costs and prices increase. The fall in the natural level of output strengthens the increase in the output gap. Natural share prices follow the natural level of output, and the natural rate increases, while the central bank substantially cuts the interest rate (more than 11 basis points) due to the downward pressure on the exchange rate. The negative real interest rate gap, in turn, leads to a further increase in the output gap, the stock-price gap, and the price indexes as shown in the figure. In addition, the sharp increase in stock prices leads to an even further increase in output through the consumption-wealth channel. The very strong and highly significant response of the monetary authority is compatible with the findings of Fig. 4, illustrating that foreign demand is one key driver of the nominal interest rate.

4.5.3. Cost-push shock

The responses after a cost-push shock are given in Fig. 7. Note that the natural values of output, stock prices and the interest rate are unaffected, and stock-price and output gaps solely reflect the reaction in the actual levels. On impact producer prices increase, and so do consumer prices. The implied appreciation of the terms of trade puts upward pressure on the exchange rate and the central bank tightens monetary policy. While the increase in the interest rate brings prices back into equilibrium, it is accompanied by a fall in output and consumption, accompanied by a fall in employment and wages. The interest rate response has a small and insignificant negative effect on share prices (reaching a maximum deviation of only 0.1% in the second quarter), which seems to be negligible.

4.5.4. Stock-price gap shock

Remember we added the stock-price gap shock to deal with the difficulty of replicating high volatility in stock-price dynamics driven largely by nonfundamental movements or factors that we do not model explicitly. As Table 4 suggests, the responses in Fig. 8 show a highly volatile reaction (the posterior mean deviates more than 11%). As the shock has no influence on natural values, stock prices and the stock-price gap show the same dynamics. To eliminate the effect on output, i.e. upward pressure on the terms of trade and the exchange rate, the interest rate must be raised by an amount necessary to keep the inflation rate, the interest rate must be raised by an amount necessary to keep the exchange rate, the interest rate must be raised by an amount necessary to keep the exchange rate, the interest rate must be raised by an amount necessary to keep the exchange rate, the interest rate must be raised by an amount necessary to keep the exchange rate.

40 Under the assumption of \( \sigma = \xi = 1 \) the foreign demand shock would have no influence on natural output at all, since all goods are assumed to be perfect substitutes for each other and \( \gamma_r = 0 \). The effect of an increase in foreign demand would have only an effect via the terms of trade.

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output (and inflation) in equilibrium. The real interest rate gap perfectly balances the effect of the stock market boom and prevents any effect on demand. The response of the interest rate is about 7 basis points at impact and clearly significant, showing that stock markets considerably influence monetary policy. Overall, the impulse responses verify our previous findings. The key drivers of the nominal interest rate are foreign demand and non-fundamental movements in share prices. Comparing Figs. 6 and 8 indicates that interest rate cuts after an increase in foreign demand are even stronger than the tightening in monetary policy after a stock-price gap shock. Cost-push shocks seem to be of less importance for the central bank under a fixed exchange-rate regime, but have a strong effect on prices.

5. Conclusions

This paper examined the business cycle fluctuations in Hong Kong through a DSGE prism and identified the macroeconomic implications of various structural shocks. Using a small open-economy DSGE model based on contemporary economic theory, our Bayesian estimation results point to positive and significant wealth effects from stock markets on consumption and business cycles. Our results also provide clear evidence, that unexpected movements in share prices have a significant and substantial effect on monetary policy in Hong Kong. While productivity disturbances are the key driver of cyclical variations in output and prices, non-fundamental movements in the Hang Seng index and foreign demand shocks largely explain short-run variations in the nominal interest rate.

Although our model is rather general, we restricted its focus to a setup with stock markets and purposely disregarded the role of shifting property prices. This omission was deliberately in an effort to limit the model’s complexity. While we believe this to be a reasonable first approximation, this modeling may well miss effects coming through the housing price channel, offering a potentially valuable subject for future research. It is our hope to be able to report some of our progress along these lines in the near future.

Acknowledgements

We would like to acknowledge the support of colleagues from the Hong Kong Monetary Authority (HKMA) in collecting the data. Furthermore, we would like to thank an anonymous referee for helpful comments on an earlier draft. Finally, we would like to thank ACE 2009, MMF 2009 and ICMAIF 2010 conference participants for helpful suggestions. The usual disclaimer applies.

Appendix A

A.1. Steady-state and linear equilibria

Following Eq. (26) a zero-inflation steady state yields

\[
1 = \frac{\gamma \Omega}{(\Sigma-1)(1+r)} + \frac{(1-\gamma)\Sigma}{(\Sigma-1)(1+r)} - \frac{\gamma \Omega}{(\Sigma-1)(1+r)} + \frac{1}{\beta(1+r)}
\]  

(55)
Furthermore, Eqs. (26), (25), (38), the production function, equilibrium on the labor market and Eq. (37) define the following steady state:

\[
\begin{align*}
\Omega &= Q + D + \int_0^1 X R'\left(Q' + D'\right) dt, \\
Q + D &= (1 + \tau)Q(1 + \epsilon), \\
Y &= AN, \\
MC &= (1 - \tau) \frac{W}{A P H} = \frac{1}{1 + \mu}, \\
D &= \frac{\mu}{1 + \mu} P_{y} Y, \\
\end{align*}
\]

where we used the optimal subsidy \( \hat{\theta} = \frac{\mu}{1 + \mu} \) as in Galí (2003).

A.2. Linear approximation

Next, we strip the model of its nonlinearities by linearizing the model at the steady state. Log-linearizing Eq. (26) using Eq. (13) follows exactly the steps in Nisticó (2005) and yields

\[
c_t = \frac{1}{1 + \Psi} E_c t_{t+1} + \frac{\Psi}{1 + \Psi} \hat{q}_t - \frac{1}{1 + \Psi} (r_t - E_t n_{t+1} - \hat{\mu}).
\]

Substituting consumption with output, using the relationship between the terms of trade and the output differentials and between the CPI and the terms of trade, gives

\[
x_t = \frac{\alpha}{1 + \mu} E_x t_{t+1} + \frac{\Psi}{1 + \Psi} \hat{q}_t - \frac{1}{1 + \Psi} (r_t - E_t n_{t+1} - \hat{\mu}).
\]

where

\[
\begin{align*}
\tilde{r}_t^n &\equiv \rho + \left[ \pi_{\alpha} \rho_{\alpha} + \Psi - \Gamma_0 \right] \Gamma_0 \theta_t \\
&\quad + \left[ (\pi_{\alpha} \rho_{\alpha} + \Psi - \Gamma_0) \Gamma_{\alpha} + \Theta \alpha_{\alpha}(\rho_{\alpha} - 1) \right] \alpha Y_t^\mu. \\
\end{align*}
\]

and \( \Gamma_0 = 1 + \Psi - \alpha \theta_\alpha \alpha, \quad \kappa_\alpha = \lambda_\alpha (\alpha \alpha + \Psi), \quad \lambda_\alpha = \left( 1 + \epsilon - \hat{\mu} \right) \left( 1 + \phi - \hat{\mu} \right) + \alpha \delta_{\alpha}, \quad \tilde{q}_t \equiv q_t - q_t^\alpha. \)

Rearranging Eq. (57) gives

\[
D = \frac{1}{1 + \tau} = 1 + \epsilon - \frac{1}{1 + \epsilon}.
\]

and as a consequence the pricing equation is given by

\[
q_t = \frac{1}{1 + \epsilon} E_q t_{t+1} + \frac{1 + \epsilon - \hat{\mu}}{1 + \epsilon} E_{d_t} + 1 - (r_t - E_t n_{t+1}).
\]

Log-linearizing Eq. (37) leads directly to

\[
d_t = y_t - \alpha s_t - \frac{W_t}{P_t} m_{c_t},
\]

where we used the assumption of a linear production technology. Using Eq. (60) with the production function and marginal costs yields

\[
\frac{W_{t}}{P_{t}} = \frac{W_{y}}{A P_{y}} = \frac{1}{\mu}.
\]

implying a relationship between real dividends, the terms of trade, the output gap, and potential output:

\[
d_t = \tilde{y}_t^n - \alpha s_t - \frac{\psi + \mu}{\mu} x_t.
\]
Plugging this into Eq. (65), and following the same steps as described in the appendix of Nisticò (2005), adjusted for the terms of trade, we derive

$$\hat{q}_t = \frac{1}{1 + \epsilon} \hat{q}_t + \frac{\lambda_{\eta}}{1 + \epsilon} E_t \eta_{t+1} + 1 - \left( r_t - E_t \eta_{t+1} + 1 - r_{t+1} \right),$$  (69)

where $\hat{q}_t = q_t^p - q_t^m$ and the natural stock price evolves according to $q_t^m = y_t^m$. Adding a shock to account for non-fundamental disturbances gives Eq. (50).

References


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